“Modeling” in Mathematics Education:

A Historical Encounter with Mathematics, Ability and Body

By

Ayşe Yolcu

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The dissertation is approved by following members of the Final Oral Committee:
Anita A. Wager, Associate Professor, Curriculum and Instruction
Thomas S. Popkewitz, Professor, Curriculum and Instruction
Dawnene Hassett, Associate Professor, Curriculum and Instruction
Erika C. Bullock, Assistant Professor, Curriculum and Instruction
Paola Valero, Professor, Mathematics and Science Education

Abstract

The purpose of this study is twofold. First, it explores how "modeling" has become a way to do science in the contemporary mathematics education, assembling with larger social, political and scientific transformations since 1950s. Second, it brings the onto-epistemological regime of school-mathematics into question by historically encountering with mathematics, ability and body and comes up with a spatiotemporal configuration, "mathematically able bodies", where mathematics appears as a cultural-historical practice in the making of able bodies.

Analyzing specific mathematics education practices in the United States during pre-post Second World War years (1930s-1945s), right after entering the curricula as a required subject, and contemporary (1980s-present), this study makes the continuities and discontinuities visible in the formation of self and society. While the "reason" of school-mathematics is historically re-iterated, as a cultural-historical practice to regulate and control people's actions and participations and to create differences, it never repeats exactly same processes but spirally extends and makes the power relations more active and diffuse.

Contemporary mathematics education becomes a model-driven science but simultaneously produces cultural norms, fabricates and divides people by classifying differences on a hierarchy. This mechanism does not position self and other in two opposite poles but reveals a process of exclusive inclusion, requiring paradoxical belonging and mutual constitution in an illusory unity, as signified in the statements like "mathematics for all". Although current "inclusive" efforts disagrees with the previous practices such as tracking, the continuities in the onto-epistemological frame reveal that "reformed" practices are the re-enactments of the Enlightenment reason and rationality, moral qualities of life, and civilization-colonization processes. The historicity of

mathematically able bodies, nevertheless, brings the porous characteristics of "knowledge" and the contestable qualities of its "nature" as a practice of change.

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Chapter I

Introduction: Statement of Problem and Questions

1. Hopes and Fears in Mathematics Education

School I

Think about a school where students' lives are strictly structured and disciplined in its quiet corridors. In this school, mathematics teachers usually transmit knowledge of different mathematical procedures using the chalkboard. They get children to practice these procedures individually. While not mandated, all teachers utilize the same pedagogical approach in this school. The act of repetition and memorization are the keys for learning mathematics. Covering necessary mathematical content of the curriculum is the main concern of teachers. Children are expected to work on the individualized booklets, then the formal textbooks. Before completion of middle school they are ranked and grouped based on their mathematical abilities. They are well behaved and disciplined during the lessons. The classrooms are quiet. Children are evaluated by their time on task, these assessments are evaluative as there is a tracking system in this school.

School II

Think about another school that is known as its commitment to progressivism, has attractive site and calm atmosphere. While children are not explicitly ordered, there is a noticeable absence of running, screaming or shouting. Their independence and responsible actions, including for their own learning, are not a result of school rules, but they can see a reason or rationale to act in this way. Mathematics, as well as different subjects, is taught with a project-based, problem solving approach having fewer consultancies of textbooks in mixed-ability classrooms. In these projects, children explore their own ideas though connecting mathematical knowledge to their everyday lives and they are given a certain degree of choice for the solution strategies. Teachers have relaxed approach for curriculum coverage and their assessment is informative, broad and holistic rather than evaluative. They have high expectations for all students and provide meaningful learning opportunities through collaborative works.

These two schools, where the narratives are adapted from the descriptions of traditional school and the progressive school in Boaler's (2002) study, suggest seemingly distinct approaches for mathematics teaching and learning. The fear of mathematics education is the first one, the "traditional", which requires discipline, ranking, individualization and standardization and so on. Everybody, including but not limited to students, teachers, parents or researchers, knows that these kinds of "traditional" classrooms have failed. An abrupt break from the "traditional" is inevitable. Mathematics education should be reformed and tracking has to be eliminated. Contemporary mathematics education reforms hope for the second narrative, the "reformed". Fair learning opportunities, a degree of freedom, autonomy, collaboration, group work and accountability have become the desired practices in mathematics classroom rather than an external administration, evaluation or ability grouping.

Historical Parallels: Emergence of School-Mathematics in the Second World War Years

Continuous effort to improve mathematics education has been commonsense for a long time although the beginning of the 20th century could be remembered as a time of intense difficulty, trouble and danger (Stanic, 1986). Almost one hundred years ago, mathematics educators were worried about the extinction of mathematics as a required subject in the curricula (Hedrick, 1936). For that time, it is hard to say that mathematics has always been a part of the school curricula as a required subject for everyone in the United States. While mathematics enjoyed a lofty status in the West as one of the disciplines of the liberal arts and a tool for mental training (Kliebard & Franklin, 2003), it was not the case for the school curriculum until the WWII years. At the period of crisis, different curriculum interest groups such as humanists, developmentalists, social meliorists, and social efficiency educators offered various justifications for teaching mathematics for all students (Stanic, 1986). Nonetheless, the emergence of school mathematics for everyone and the calls for change should be read as part of the broader historical transformations that arrange the social and spatial configuration of everyday life and the society.

During the pre-post WWII years, as in contemporary practices, the reform and change in mathematics education and curriculum was necessary, but the change was bounded by the given fact that "mathematics should be an important part of school curricula" (Stanic, 1986, p. 193). The presumption that mathematics as a part of daily life, in fact, carried the Cosmopolitan notions of enlightened citizens embodying particular, in this case mathematical, modes of seeing and acting in the world (Popkewitz, 2008). Mathematics embedded in the modern life was bounded to modes of living as rational, intelligent and efficient citizens. This assemblage produced cultural theses for the mathematically able bodies to order proper and improper modes of living, what it means to be a modern citizen and who were outside of that configuration, in need to be rescued.

At the turn of the 20th century, the conventional history of mathematics education talks about a period of unrest and crises for the mathematics education community where there were several unsolved issues for the place of school mathematics. Mathematics was an elective subject beforehand, except the primary school. The low number of enrollment in mathematics courses was alarming the mathematics educators. The steady decrease was not limited to enrollments in algebra and geometry courses but also their requirements (Stanic, 1986). It was both a matter of quality and quantity. In addition to this, a survey of 416 secondary school teachers (including 48 mathematics teachers) conducted by George Counts, revealed that all teachers thought that their subjects should be maintained in the curriculum except the teachers of mathematics. Some of the mathematics teachers thought that fewer pupils should take mathematics classes (p. 196). There was no reason to study mathematics except fulfilling the college entrance requirements. However, at the same time, the college was not that desirable for the youth as an effect of economic depression and war.

The question was what should be the contribution of mathematics to general education for all children? The shifts in enrollments in mathematics courses, industrialization and influx of European immigrants brought new urgency to the ideas such as applicability and usefulness of mathematics in everyday life of citizens (Progressive Education Association [PEA], 1940). The "major aspects of living" and "democratic ideals" had significant effects on what made school mathematics school mathematics (pp. 73-74); however, the war also had a vitalizing effect on mathematics in terms of using it in quality control, technologies for the army and communication, which also had extensive peace time uses also. The result for mathematics and science in schools was to return their prestigious positions that had not been recognized for several years. Nonetheless, it was not merely returning to the old days of educating particular groups of student. Rather, the primary purpose was to provide the detailed coordination of a complex operation involving many people, mathematics and its applications. The mathematics education of the post-war America had to be planned in advance and arranged towards not only functional competence of the population but also the scientifically oriented society (Jones & Coxford, 1970, pp. 58-60). Although much has focused on the progressive era reforms and post-Sputnik national policies for school mathematics, the war itself and immediate post-war years need to be attended considering the multitude of documents, reports or policies written for school mathematics following its precarious position in the curricula.

In the broader scale, indeed, these statements and practices set the ground for the recurring themes in the history of mathematics education and the legitimizing reasons for the place of mathematics in the curriculum (Garrett & Davis, 2003). Formulation of the new objectives centered on the physical universe, society and the child initiated the different curriculum outlines in relation to the demands of "ordinary life", "leadership and higher culture" and "specialized use". The scientific and technological nature of the modern warfare removed the doubts about the place of mathematics taught and learned in schools. The importance of school mathematics became unquestionable as its utility on the war technologies was broadened to the industrial work to contribute the war effort. In other words, a curriculum issue was intensified toward a "patriotic duty" where the national survival depended on not only the fights in the battleground but also the efforts in the home front and classrooms (pp. 497-500).

These rationales were going to be extended to the peacetime use and set the ground for studying mathematics in schools. Once the decreased quality and quantity of mathematics courses in schools were going to shift in an unquestionable and undoubted status among the school subjects for progress, development, civilization, democracy, justice and efficiency.

Since then, mathematics education in the United States has been faced with various reform initiatives. Moving mathematics teaching, learning and research practices forward has become the greatest hope for mathematics educators whereas the fear is the traditional mathematics, which should be abandoned. The reform movements in mathematics education, however, continually express the hopes for a Cosmopolitan society and child, which resonate with the Enlightenment's hopes of the world citizen committed to ideal values about humanity and the fears of degeneration and decay (Popkewitz, 2008). For example, how to act and participate are characterized through the rationales of mathematical modes of thinking for progress and development. The idea of using mathematics and mathematical relationships for everyday life, in fact, embodied the cosmopolitan principles of reason such as problem solving or collaboration in the learning communities for its enlightened citizens while simultaneously invent those deviated from this reason. For all of these reasons, I choose this period to historicize with the notions of "mathematics", "ability" and "body".

Back to the Contemporary: Standards of Mathematical Practice

Mathematical modeling has emerged as a "new" mathematical practice for the "mathematically proficient" student in the most recent standards (National Governors Association [NGA] Center for Best Practices & Council of Chief State School Officers [CCSSO], 2010). For this mathematical practice, students are expected to apply mathematics to understand and address the problems of daily life depending on their "mathematical maturity" (p. 8) and "confidence" (p.7). Identification of important quantities in a practical situation entails a particular style of reasoning about the world where students are asked to use tools such as diagrams, two-way tables, graphs, flowcharts and formulas to "tame the chance" (Hacking, 1990). In one of the modeling activities, to illustrate, children are expected to identify the number of fatalities per year, per driver and per vehicle-mile traveled to find "a good measure of highway safety" (NGA & CCSSO, 2010, p.58). What being produced here is the pre-emptive realities for the unknown future and actionable practices for the present (Amoore, 2013), decided by mathematical models and quantitative relationships through the notions of safety and risk. In other words, a social phenomenon (i.e. highway safety) is produced through the "possible events" (Foucault, 2007) identified as data points such as fatalities per year or per driver. Obviously, these projects do not only intent to develop an understanding of mathematical content but also the standards of participating in daily life. The concerns for safety and risk resonate with the redemptive narratives of American exceptionalism where the pursuit of happiness is the goal and also in the line with cosmopolitan ideals such as harmony and stability (Popkewitz, 2008). Normalization of particular modes of life via appropriate risk analysis simultaneously generates and targets those "unlivable", "insecure" and "unhappy" cultural spaces (since mathematical models do not foresee those or they are too risky). What I argue throughout this study is the quantification of life and mathematizing the natural or social phenomena do not only entail teaching and learning mathematical knowledge but also produces *mathematically able bodies* ordered and regulated by mathematical modes of thinking, seeing, acting and participating in the world.

Current mathematics education reforms are offered as an innovative resource for "systemic excellence" that will bring development and progress for all children and the society (NCTM, 2014). New methods of teaching and curricular ideas are presented to ensure the high quality of mathematics for all and to make a difference. Yet, in this study, I am interested in the making of difference; that is, how mathematics education plays an active role in "making up" (Hacking, 2007) mathematically able bodies.

Those who think that standards do not go far enough also utilize mathematics as a tool for empowerment, voice and liberation or as an instrument of power that ultimately reconstruct an enlightened, democratic and just society (Apple, 1992). In the counter movements against the standards, the mathematical practices for the students, who are at the same time presumably the change agents, entailed similar processes. For example, in one of the social justice mathematics problems, children are expected to identify the number of people who applied for mortgage loans and the number of people who can or cannot get the loans in order to identify whether racism or income is a factor for getting mortgage loans (Gutstein, 2006, pp. 58-61). According to Gutstein, what makes this "real-life project" most "engaging" is the incorporation of contextual elements from students' lives. Home ownership is connected to daily life of students and it refers to stability, security, and some prosperity (p. 60), which in fact ensembles with the American notions of progress and happiness. Through these mathematical modeling processes and mathematizing "daily life", students do need to use mathematics not only to understand their conditions for living but also challenge them to make these conditions better, more stable and secure. The trigger for moving society and mathematics education practices toward an equitable system and prosperity for all is connected to the cosmopolitan notions of reason such as problem solving or collaboration in the learning communities for its enlightened citizens. The hope for mathematical understanding of these social justice issues invents those who resisted engaging in these real-life projects. As Gutstein (2006) reports, "several students were inconsistent, resistant, or sometimes confused [when] trying to make sense with mathematics of complicated social phenomenon" (p. 69). A similar, not the same, process of double gestures, hopes and fears in these projects, operating here and it makes up different kinds of mathematically able bodies and their others assembled with the premises of the projected pedagogy. What remains unquestioned is the authority of school mathematics that is crucial for constructing rational and reasonable arguments to analyze a social situation and for inclusion of those excluded to learn mathematics before.

In the contemporary narratives, the emergence of the new roles for children such as problem solver, decision maker, collaborator or change agents are the products of various discursive and scientific mechanisms along with contingent historical and political forces. In order to challenge these long held understandings that produce these kinds, in this dissertation, mathematics itself is not treated as a natural, sacred or given thing. Rather, mathematics, as a discursive formation of multiple practices, is brought into question and is historicized to get a sense of how these multiple practices become possible to order, differentiate, classify and normalize kinds of mathematically able bodies and their others.

2. Mathematics in Schools

Mathematics in schools is not one but many: a vehicle for social efficiency (Garrett & Davis, 2003), a mental discipline (Philliphs, 2015), a competency for democratic citizens (Romberg, 1998), a cultural representation against the Eurocentricity and masculinity (Bishop, 1990; D'Ambrosio, 1985), a formatting power to shape the society (Skovsmose, 1994) or a weapon to change the society (Gutstein, 2012). It is for rationality, citizenry, social efficiency, cultural representation, democracy, social justice and so on. What this plurality might suggest that there is not a one form of mathematics in schools at all, or its products are many. Nonetheless, in mathematics education research practices, flagging "mathematics" as a key for future economic and professional opportunities as well as for solving the problems of social justice, oppression, poverty, sexism or racism is prevalent (Pais & Valero, 2012, p.19). This situation is not limited with the research practices but also permeates the everyday life. The quantities are considered as part of daily life in modern societies. It has, somehow, become awkward to think of a person without certain knowledge of mathematics. Throughout this study where the question of mathematics is central for me, I shall be interested in mathematics but not through the questions as like "what is math?" or "where is math?" rather, re-iterating Hacking's (2011) question: What makes school mathematics school mathematics?

While my intent is not to unearth an essential or a stable meaning of "mathematics" or to explore an inner truth for what constitutes mathematics teaching and learning, as if there is, I shall be discussing how various kinds of answers woven into the cultural field of mathematics education as technologies to order and regulate the pedagogical space that construct their objects, only in their temporalities. As Popkewitz (2008) argues, thinking of mathematics as a field of cultural practices "is a way out of the controversies that divide education into realist and antirealist camps, the unproductive separation of epistemology and ontology, and the division between subjectivist and objectivist worldviews" (p. 148). In this sense, it is possible to contend that the mathematicss exist but they are contingent upon the style of reasoning, truth statements and the historicity of the subject matter in a field of multiple discursive practices. Thinking mathematics and mathematics in schools as a field of cultural-historical practice, also, troubles the binaries that constituted through the division between whether mathematics in the nature is experienced by humans or mathematics is determined by the human mind.

Although the discussion above might give some sense for the problematization of school mathematics and the need for rethinking of it as a field of cultural practice, for the mathematics education, several questions remain. For example, if mathematics is "pure", "abstract" and "decontextualized" knowledge, then what do social categories of people (i.e. efficient, productive individuals) have to do with mathematics? If mathematics is a mental discipline, then how does it become the solution of justice problems? How could we explain these different categories and various modes of mathematics in schools? How is it possible for mathematics to produce different "kinds of people"? What we have, then, is the transmutation of subject matter knowledge into a particular mode that is regulated by the principles of social and psychological sciences along with the historical and political conditions that shapes and fashions participation and action in social life, what Popkewitz (2004) calls "alchemy". According to him, alchemy functions as a tool to explore the limits of didactics through the "cultural theses" produced for school subjects in a network of inscription devices and intellectual tools that translate the school subjects and construct its objects and truth statements (Popkewitz, 2008).

The diagnosis and analysis of inscription devices, intellectual tools, historical-cultural logics and comparative reasoning of cosmopolitanism does not entail a network of separate actors, as if they could precede one another, but it is a field of cultural practices that is constituted through entangled relations between and within them, which I shall refer throughout this study as the "discursive assemblage of school mathematics". That is, we do not have a transformed subject matter through pedagogical inscriptions preserving its ontological status. Instead, there is a metamorphosis of mathematics, rather than a translation as if there exists an essential meaning of it, which makes mathematically able bodies possible. Mathematics is becoming a different entity that could be understood by looking at the entangled relationships within its discursive assemblage. The discursive assemblage of school mathematics, then, might be somewhat similar to "rhizome", to borrow from Deleuze and Guattari (1987), consisting of multiplicity of lines, segments that are connecting heterogeneous ways with non-hierarchical entry and exit points. What is at the issue to consider the discursive assemblage school mathematics as a whole that enable different technologies or apparatuses to be intertwined in rhizomatic ways, which is not a generative or reproductive model for tracing structures. Rather, the assemblage is a map, which is open to constant modification (pp. 12-25).

In our contemporary world, the social life is regulated through mathematical calculations; weather forecasts, highway safety, calorie diets or crime rates, to name a few. It has, somehow, become commonsensical to act and participate in the daily life with the knowledge of mathematics. The will to provide a quality of mathematics is also prominent in mathematics education research as well (Pais & Valero, 2012). The commonsensical importance of school mathematics has been examined before through its repressive functions: as a use-value in capitalist societies for credibility and marketability (Pais, 2013), as the sublime object of ideology in mathematics education practices and the source of the problems of equity (Pais & Valero, 2012) or as an apparatus of social reproduction to track children in early grades (Stinson, 2004). What remained unquestioned, however, are the productive aspects emerged from the network of practices not from the mathematics itself. That is, the interest here is about how discursive assemblage of school mathematics is making kinds of people not because there is an inherent power in itself but its connections with exterior elements to order and regulate proper and improper modes of life. That is, how the school mathematics is making mathematically able bodies and their others through embodiment particular modes of living, action and participation in personal, social and political spheres of their lives.

This study diverges from and goes beyond the previous analyses in following ways. First, school-mathematics is not merely positioned as an "object" of ideology, as a reproductive "apparatus" or as a "thing". On the contrary, as explained before, school-mathematics is an assemblage of discursive practices, consisting multiple pedagogical devices, intellectual tools, cultural logics, historical-political conditions and so on. None of them precedes one another; only the entangled relations can give school mathematics a meaning or some sense of existence. Second, the concept of power is being rethought throughout the study. Agreeing with Foucault (1975), power is not a privilege or something to be possessed; rather, power is exercised and it produces a reciprocal relation with the knowledge. As he further explains, "there is no power relation without the correlative constitution of a field of knowledge, nor any knowledge that does not presuppose and constitute at the same time power relations" (p.27). The purpose of this research is, overall, to investigate how the various discursive-material practices and intertwined power-knowledge relations make the school-mathematics significant in undoubted ways. It rethinks how inclusion and exclusion issues occur by studying the making of differences. Throughout the study, answers for the following questions is sought:

(1) How might the commonsensical practices in mathematics education be rethought when knowledge is considered in productive terms within the discursive assemblage of school mathematics?

(2) How might inclusion/exclusion issues in mathematics education be rethought when knowledge is considered in productive terms actively involved in the making mathematically able bodies within the discursive assemblage of mathematics?

3. Corporeal Regulations in the Hopes of School Mathematics

Let's go back to the narratives of "reformed" and "traditional" mathematics. In general, this change is considered as revolutionary or as a "historical turn" in mathematics education. When closely examined, it is also possible to see the shift in cultural theses produced for the children such as problem solver, autonomous individual, decision maker, and collaborator. Nonetheless, school reforms play an important role to cultivate the inner qualities of child and order (im)proper modes of life for the self and society (Popkewitz, 2008). Although the cultural field of mathematics education is not directly disciplined and strictly structured in most cases of present practices, the new regulatory mechanisms are built in order to control the life. For example, in the narrative of the second (reform) school, the development of student independence is encouraged while children are controlled by themselves to act responsibly both in terms of their social lives and their own mathematical learning in or out of school environments. These children are not ordered or ruled by their teachers and administrators, but they have developed their own "reasons" to control their own actions. As Deleuze (1992) points out, in "societies of control", while disciplinary systems have undergone a crisis, the new forces are gradually instituted and different control mechanisms are built almost equal to the harshest of confinements despite the expressions of new freedom and flexibility. Having said that, disciplinary mechanisms are not left behind or disappeared. Children are being tested, ranked and classified as "mathematically proficient" and they might experience differential learning opportunities where the mathematical ability can be considered as a "marked category" for their bodies (Damarin, 2000). Therefore, the critical analysis of the discursive assemblage of school mathematics requires an examination of two aspects of productive power operating in these two spheres through its multiple effects.

The regulation of social space, as Foucault (2007) would argue, requires new governmental rationalities that control the life of human beings. These effects are neither equivalent nor an extension of one another and they are not completely distinct from each other, but they are assembled in the complex array of power-knowledge relations. What being produced are, first, the mathematically able bodies desiring to take control of their mundane details of life from health care to politics through mastering school mathematics or some kind of knowledge. Second, categories of distinctions among those mathematically able bodies are formed. The double gestures operate in two spheres: about promising futures for individual's own well-being and about "educating" kinds of people who willingly take care of their personal, social and political lives. Put it differently, mastery of school mathematics becomes a "norm" for the bodies (i.e. children) for their own benefits. At the same time, the organization of pedagogical space for teaching school mathematics in this particular way becomes a norm for the making of mathematically able bodies and produces the categories of distinctions and differentiations.

The seduction of mathematically able bodies has some historical parallels. The habit of using mathematics in everyday life in order to be and act as "efficient" and "intelligent" citizen, for example, can also be traced back to the 1930s-40s, right after entering the curricula as a required subject for all students, to prepare them for life. Modernization of daily life, at that time, required "possession of knowledge" that make citizens act rationally where mastering mathematics was seen essential to understand machines, housing, business, transportation, public and personal health, investment, insurance or taxation (National Council of Teachers of Mathematics [NCTM], 1944, p. 227). What the analysis of the hopes and fears in the statements and practices indicate is that, there is something in the reason of the discursive assemblage of school mathematics that fabricates the mathematically able bodies and excludes their others and put them into different level of tracks. Nonetheless, the differentiation of mathematically able bodies and their others did require different kind of technologies to "see" the this distinctions at these particular historical moments, which would not show an accumulation of tools and their improvement within time rather the invention of new strategies to secure the power relations.

Different historical and political conditions, techniques, technologies, calculations and forms of knowledge come together in entangled and indeterminable ways to think school mathematics as an "essential" life skill and as a marker of difference. The configuration of nation states, formation of modern selves, changes in the political economies, scientific and technological developments, the practices in educational sciences, to name a few, invented these bodies that can mathematically "function" in their personal, social and political lives. What I am interested in, through analyzing these mechanisms of power, is the politics of truths embedded in the discursive assemblage of school mathematics that govern the conduct of people in multiple spheres of life. Governmentality requires the invention new technologies, as Rose (1999) puts it, to shape, order and fashion the conduct of people to manage the economic life, the health and habits of the population, the civility of the masses and so forth (p. 18). Rather than asking why this happened and looking for the origins, my questions are: How these multiple effects happened? How has it historically become possible to think school mathematics as an "essential" life skill and as a demarcation between bodies? What scientific rationalities and strategic technologies overlap with the historical conditions that make mathematically able bodies and the differential constitution from their others possible?

4. Organization of Chapters

My analysis focuses on two moments in the history of mathematics education practices, which are the pre-post WWII (1930s-1945s), right after entering the curricula as a required subject, and today (1980s-present). This exploration reveals three planes where the entanglements get intensified. First, there are parallels in the making of self and world through a stabilized category of mathematics. Second, the developmental narratives on (mathematical) ability, thinking, and reasoning are never disrupted. Third, there is a question of population in both of these moments, concerned with the adapting (math) instruction or curriculum to meet the "diverse" bodily needs and interests. I historically analyze these planes, without taking them for granted, to make visible how a particular child is fabricated as mathematically able. In the contemporary practices, different than the past, we shall see an emergence of a particular mode of thought; that is modeling. Modeling, as a way to do science, in mathematics education holds things together and make the future a category to be acted upon. Child constructs (mathematical) models for the world or space. Teaching becomes an act of modeling mind and an "anticipatory thought experiment" for the regulation of pedagogical space. Modeling mathematics instruction is to "manage the diversity of student work" to make the teaching more efficient and productive. Although these practices are different than from the "past", there are important parallels along the line. What become convergent across the historical moments are the practices producing objective knowledge and practices objectifying particular kinds of people. In what follows, I will give a brief synopsis of these and the chapters in the rest of this dissertation.

In the next chapter (chapter 2), I present the theoretical orientations and analytical strategies of this study. All these theoretical insights are not provided with a purpose of projecting them onto the mathematics education, particularly to this study about school mathematics. Instead, my intention is to pursue the question of school mathematics and rethinking inclusion/exclusion issues in mathematics education with different theoretical toolkits without considering them as deductive sets of operations to test the correctness of things and words.

Chapter 3 is about mathematics. Across the two moments I analyzed, what have remained similar, even continuous, are the notions of "mathematical precision", "accuracy", and "mathematical clarity". However, this is less related to mathematics as a subject matter but more of a civilization process of social and scientific practices, which has to with the moral character of doing science to find a correspondence between mathematical practices and the real world rather than a theoretical rigor. In the past, the world was thought to be inherently mathematical and the task of man was to discover it. However, today, there is not this presupposition. The child can mathematically model a real situation that is not inherently mathematical. So, I will be tracing the changes in the practices that made the shifts possible from "discovering the mathematical world" to "mathematical modeling of the world". Situating the "mathematical modeling" in the broader historical transformations such as shifts to rational choice, risk-based security calculations, system theories in social and natural sciences and anticipatory politics of knowledge will help me to understand how school-mathematics is a cultural product of these conditions. Yet, school-mathematics is also producing the culture and also particular kinds of people. In modeling activities for children, researchers talk about "translation" of this scientific practice of modeling for "ordinary people", which requires them to talk about the standards of participation in those modeling activities such as mathematical argumentation, taken-as-shared knowledge or collective norms, as the regulation of pedagogical space. In these practices, the double gesture of schooling made the normal and deviants possible. In tying them altogether, at the end of this chapter, my aim is to understand the difference is produced through the amalgamation of distinctions not because of a particular thing.

In Chapter 4, I trace the developmental narratives about mathematical ability. I look at how "mathematical maturity" was historically possible at that time and how it transmogrified into "learning progressions" or "learning trajectories" now. What is the "science" that makes these constructs possible? Once, it was the experimental designs with control groups, now it is teaching experiments where they do presumably "push the limits of dichotomy between process and content". In this chapter, I historically situate these emergent practices (i.e. teaching experiments) to understand how they become possible. In these contemporary practices, we shall see how the cybernetic rationales, system theory and anticipatory logics came together and make these "anticipatory thought experiments" possible to "design a teaching model". Researchers become the teachers and they conduct these anticipatory thought experiments to *model* the minds of children. Although the "teacher ability was an irrelevant factor" in the controlled experimental designs, teachers are the inquirers of their classrooms where they analyze the practices to "adapt, test, and modify the sequence" of mathematical activities in the contemporary practices. I trace the legitimizing reasons to understand how these shifts become possible and politics of knowledge embedded in these practices and how the differences are constructed and put in a "developmental" order then and now.

Chapter 5 is about a continuous trend in mathematics education, which is about meeting the needs and interest of children through accommodating/adapting the instruction. My analysis across these two moments in the history of education revealed the prevalence of the human needs and interests. In this chapter, I want to consider how school-mathematics, becomes historically possible as human need in terms of governmental practices of state rather than assuming humans inevitably need mathematics by their nature. Looking closely, the projects of school-mathematics are not only concerned with mathematics as content, but these were more about the managing and controlling the human conduct in the daily life with particular practical/useful knowledge through the "development of desired characteristics of personality". However, the practices differ across these moments. In the past, the solution was to track children to meet their "diverse" needs and interests into social mathematics courses to produce intelligent and efficient citizen. However, today, we see the emergence of *modeling* the instruction of mathematics through "an efficient use of students' responses by teachers". The anticipation of diversity of responses is to maximize the effectiveness of teaching through a model of "how-to package" for instruction. This long-held notion of efficiency is also tied to American culture and Dewey's pragmatism. Another dimension of needs and interest is to set mathematics as a tool for democracy. While this was much more laudable in the documents and research reports around the Second World War, it is still possible to recognize this dimension of school mathematics in the contemporary practices in the making of self-ruled citizens or individuals to maintain the social stability or consensus. However, the technologies that make possible these ideas are shifted. Today, they are pre-emptive and precautionary with the self-assessment strategies or cultivation of "growth mindset" rather than "habits of mind". My aim to look at how the double gesture of schooling produces subjectivities for the self and other in the name of "math for all".

Chapter II

Studying the Corporeal Regulations in the Discursive Assemblage of School Mathematics

1. Why These Investigations?

The purpose of this study is to rethink the commonsensical practices in mathematics education when knowledge is considered in productive terms within the discursive assemblage of school mathematics. As briefly introduced, school mathematics has historically become a commonsensical but an "active" actor to produce the categories of distinctions and differentiations among the kinds of people. As an effect of its cultural-historical construction, the mathematically able body becomes possible in a specific time and place, embodying particular modes of living and acting in the world. In this chapter, my purpose is to layout the theoretical and methodological grid for this study and the motives for why we need a different kind of analysis of our contemporary practices in mathematics education. Following the questions that emerged from the literature that considers the sociocultural issues and material practices within mathematics education and makes a critical analysis of the long-held understandings in mathematics education, I present the theoretical orientations and analytical strategies of this study.

Sociocultural Studies

Thinking of school mathematics in its sociocultural context and problematizing the rational mind has already been a consideration in the literature. There have been a number of studies investigating the social, cultural and political dimensions of school mathematics. For example, studying the nature of mathematical experiences and identities of individuals as they participate in mathematical activities is an emergent research area (i.e. Esmonde & Langer-Osuna, 2013; Wood, 2013). The construct of mathematical identity is considered as an extension on the research on mathematical reasoning through including how students think about themselves in relation to mathematics and their persistence, interest and motivation to learn mathematics (Cobb, Gresalfi & Hodge, 2009, pp. 40-41).

For the most part, nonetheless, this scholarship has anticipated the immobility of mathematics despite of the incorporation of contextual elements, pedagogical methods and practices of a mathematics classroom. The identities that students develop in a mathematics classroom, for instance, can only occur within the extent of students' affiliation with mathematics (Cobb, et. al., 2009). Comparing the distribution of the discussion patterns in relation to having access to "significant mathematical ideas" might give some level of detail to be investigated; however, what is taken for granted is a some kind of affiliation with mathematics is required for the construction of "normative identity as doers of mathematics" to be "an effective students" in the mathematics classroom (Cobb, et. al., 2009, p. 56). The object of research emerges as analyzing and representing the kinds of participation or affiliation structures through the students' views about and appraisals of how the classroom works. If we accept this framework, how and to what extent is it possible to resist the ontological primacy of mathematics where "students would have to identify with the role of an effective student as delineated by [mathematical] obligations in order to develop a sense of affiliation with mathematical activity as it is realized in their classroom" (Cobb, et. al., 2009, p. 63)? What makes the mathematical practices in a classroom as "significant ideas" that make students engaging or other practices that make them resisting? Are the "mathematical obligations" products of classroom practices alone? The constructed identity presumes a subject, who appropriates already existent mathematical obligations in some ways, although they could be modified, to be *an effective student*, which is already a product of the discursive practices of schooling. While the recognition of social variables in the classroom discourse might give some further understandings about how mathematical identities are socially constructed in mathematics classrooms, there always remains a tendency to treat school mathematics as passive, muted and distant subject matter. Also, the representation of mathematical participations and affiliations assumes a subject exists before the action of doing mathematics. In these analyses, a gap between mathematics and the child continually remains; at the same time, the kinds of affiliations, independent from the mathematics, become the determinants of the identities. Then, there remains a gap between this subject and mathematics. In addition to the presumed subject before the action, as Barad (2007) argues, taken-for-granted ontological distance produced between knower and known raises questions of the accuracy of representations, in this case the kinds of affiliations, which ultimately generates the questions of inclusion/exclusion: What kinds of affiliations are taken into account to be an effective student of a mathematics classroom? What other kinds of affiliations are implausible in the context of developing a normative identity as doers of mathematics?

Let us look at another study that investigates the linkages occur between students' participation in mathematical discussions and social identities (i.e. racial and gender categories). In this study, the opportunities to learn mathematics is dependent upon how the teacher is drawing from the cultural and linguistic strengths of students in a classroom, which could be understood within the context of multiple positioning of different actors such as teacher, peers or mathematics in different ways (Esmonde & Langer-Osuna, 2013). For example, one of the students is positioned having more power due to his racial and gendered identity as a White boy along with his alignment with the existing interaction patterns in the mathematics classroom. At the same time, some other students are also positioned with the power as they write a counter narrative to the deficit perspectives about African Americans and mathematics circulating in the media and research practices (pp. 309-310). While the recognition of social variables (i.e. race, gender) and various power dynamics in mathematical activities are important, making children's experiences visible in mathematics classrooms and represent them in research settings prevent a critical examination of how these "differential" experiences historically emerge in relation to discursive formation of these categories. These kinds of analysis are limited to understand the power mechanisms that historically generate mathematics as a form of social identification. For instance, we don't know what makes the White boy positioned with more power in a mathematics classroom or what makes the deficit perspectives about African Americans and mathematics possible. As Scott (1992) mentions, representation of experiences or identities of subjects is actually a decontextualization since the critique is made outside of the discursive construction while reifying agency as an inherent attribute of individuals. For example, it has remained an inherent characteristic of African Americans to produce a counter narrative for their own positioning in the media and mathematics education research practices.

In this study and the related literature, furthermore, researchers do represent the ways of participation in the classrooms. The body becomes a tool between school mathematics and these racial or gendered ways of knowing. That is, the body appears as a passive medium, like mathematics, that cultural meanings of mathematics and teaching mathematics could be inscribed and these meanings are external to the self. For example, the White male body becomes a corporeal instrument for African Americans to practice their own counter stories on mathematical engagement through embodying already-formed mathematical practices such as conjecturing, requiring precision and detail, respecting mathematical conventions, and applying abstract mathematical ideas (Esmonde & Langer-Osuna, 2013, p. 309). When the resistance is the representation of engagement of classroom talk through these discursively formed and historically recognizable mathematics education practices (i.e. mathematical precision), there remains a "repeated inculcation of a norm" for gendered and racialized bodies, setting the boundaries for the humanly thinkable (Butler, 1993, p. 8). In this sense, I aim to question and problematize what makes these mathematical practices as recognizable and humanly thinkable while the other styles of knowing is foreclosed, erased, being refused or unthinkable.

Material Embodiments

In the scholarship on embodied mathematics, as de Freitas and Sinclair (2013) claim, most of the researchers eloquently showed that how human thinking involves various parts of the body rather than just the Cartesian mind; however, mathematics itself remains as a distant subject. They argued, as I agree, "… much of the work on embodiment tends to fix the body in simplistic terms and to crystallize the discipline or "content" into a passive role" (p. 454). That is, while there are inclusions of gestures or bodily movements into the analysis, the ontological status of mathematics are preserved, which makes it detached from the selves.

Recognizing the limitations of the distance between mathematics and the learner of mathematics, de Freitas and Sinclair propose "a new materialist ontology" studying the mathematical body as an assemblage of human and non-human mathematical concepts. The approach they adopt seeks to analyze the complex entanglement of the processes of knowing and becoming while reconsidering the dynamicity of non-visible mathematical concepts. In this way, they argue for the fluidity and instability of mathematical concepts, adapting Chatalet's philosophy of mathematics, that constitutes the new assemblages of human and non-human actors at each time. Although studying the entangled relationships between and within the body and the fluidity of mathematical concepts might provide an alternative way to think the bodily experiences of mathematics learner and to problematize the distance occurred between body and mathematics, the analysis has a little clue about the institutional discipline and control mechanisms that historically become possible to regulate the pedagogical space.

The interest in how mathematical activity is implicated in this "complex process of becoming human" (de Freitas & Sinclair, 2014, p. 57) precludes an analysis of discursive formation of kinds of people, where the mathematical concepts are not the only non-human elements in this discursive assemblage. That is, the inscription tools and intellectual devises that metamorphose the mathematics are not included in this proposed materialist ontology of mathematics. While destabilizing the concepts and redistribution of agency within different actors has potentials to rethink what is historically included and excluded in a mathematical activity, the preclusion of other agents, such as the scientific knowledge on pedagogy, that make the differences and distinctions between kinds of people possible is the interest of my project. The discursive constitution of humanly unthinkable in daily life raises several questions: What are the historical contingencies that make school mathematics school mathematics? How is it possible to think the discursive assemblage of school mathematics as a productive agent that makes different kinds of people? What are the corporeal regulations of school-mathematics in daily life? What are scientific mechanisms that produce "norms" for inclusion and exclusion within the regulatory assemblage for the bodies to live and act in particular ways? These are the questions naturally emerged, which is an interest of this study, from what they leave out. As Barad (2007) would argue, "[what] is excluded in the enactment of knowledge-discourse-power practices plays a constitutive role in the production of phenomena- exclusions matter both to bodies that come to matter and those excluded from mattering" (p. 57). In this sense, the entangled relations in the discursive assemblage of school mathematics are productive, and what are excluded in these relations matter.

(Re)imagining the Cultural Politics of Mathematics Education

In asking these questions emerged from the sociocultural studies on mathematical identities and inclusive material embodiments, my point is not to question whether or not social variables or what kind of non-human actors are included. Rather, I would like to invite readers of this study to (re)think the cultural politics of the discursive assemblage of school mathematics. How are the relations of power understood? How could we think the subjects and objects of school mathematics as a discursive formation? What are the historico-political conditions that make categories of school-mathematics possible? How can we rethink the inclusion/exclusion mechanisms without assuming the existence of autonomous subject while challenging the taken-for-granted nature of mathematics itself? At this point, I agree with Barad (2007), where she says, "any proposal for a new political collective must take account of not merely practices that produce distinctions between human and nonhuman but *the practices through which their differential constitution is produced*" (p. 59, italics original). Then, historicizing, which simultaneously brought the object of governance and their differential constitution into question, is a strategy to reimagine the cultural politics of mathematics education.

There needs a kind of analysis that questions the legitimizing practices of the discursive assemblage of school mathematics that produce kinds of people, which I have already been calling as mathematically able bodies. In this sense, my focus is not on the materiality of children's lives or their bodily experiences, nor what children did and how they lived; rather, I am interested in the discursive assemblage of school mathematics that produce the categories, creates the hierarchical ontologies and generates corporeal regulations. As an ongoing purpose of this study, what I would like to focus on how mathematics as a cultural practice historically become possible to signify a particular regime of practices to make able bodies and to govern the self and society. But, what it means to write a history for the body? How the mathematically able bodies could be understood in the discursive assemblage of school mathematics? In the next section, I aim to conceptualize what I understand with the notion of body.

2. Mapping Mathematically Able Bodies in the Discursive Field of Mathematics Education

Let us remember the cosmopolitan hopes of mathematics education; to name a few, these are using mathematics in everyday life to solve the problems, making mathematical argumentations for democracy, freedom or liberty. Let us consider, this time, not an ideal figure of "mathematically proficient" student prefigured by the standards (i.e. NGA & CCSSO, 2010), but some of the "voices" of a mathematically successful student who has developed *robust* mathematical identities (Stinson, 2013, my italics). For example, Keegan, one of the participants in Stinson's study, remembers how he hated mathematics but were aware of its necessity for success during his high school years. However, the rationale for him to persist studying mathematics was not to pursue further study in a mathematics related academic field but his recognition of the "ways of maneuvering [life] through mathematics" through his parents' experiences such as seeing life as a "huge mathematical equation" or "running a household while being Black". As a future clergyman, he tells how he enjoys doing mathematics and solving problems effectively to manage the church and to maintain its healthy financial status. While he had various opportunities to learn to celebrate his Blackness, he argues, “doing well in mathematics and academics in general was not a choice but just a precedent-set expectation” (pp. 80-83).

The reason of re-retelling this story is not to affirm his success in mathematics or represent his individual racial struggles while negotiating the multiple identities he has or represent an individual subject to affirm the multiplicity of mathematics education discursive practices. However, my intent is to map various situations and illustrations that regulate the operations of bodies in schools, or years after school as in this case, as well as many other cases. In this narrative, body is not an object or target of mathematical power. Rather, mathematical practices have emerged as meticulous control mechanisms over life. The robust mathematical identity that is developed, for example, is not reproducing an anticipated normative identity determined by particular mathematical obligations. On the other hand, the deployment of mathematics in various life contexts, which does not have to do with schools only, might suggest how the discursive assemblage of school mathematics proliferates, innovates, annexes, creates and penetrates bodies in an increasingly detailed way and controls the populations in an increasingly comprehensive way (Foucault, 1978, p. 107). In this sense, the mathematically able bodies are not a merely an inscription of the normative ideals but they are active entities emerged through an appropriation or enabling of discursive assemblage of school mathematics while re-creating it.

The dynamic, multiple and fragmented nature of the healthy mathematical identity that this mathematically successful student developed, as Stinson (2013) argues, should be best understood within the context of how he repeatedly accommodated and resisted within the unjust sociocultural conditions that already positions himself as a kind of problem (pp. 91-92). If we accept this analysis, how could we understand his love of everyday mathematics? How could we explain the use of mathematics to administer his life in an unjust society? How can we understand the discursive making of school-mathematics as a product of this "unjust society"? How does it become possible for him to maneuver his life with "everyday mathematics"? What we have, then, a form of a power that optimize the life through positive influences while subjecting bodies to precise controls and regulations, which we should situate within the broader transformations of modern societies that take the life of its citizens as an object of governance. That is, a shift starts to happen from "disciplinary technology of the body" that produces individualizing effects towards "regulatory technology of life" that control the every minute of human species (Foucault, 2003, p. 249). In these both forms of power, which are not exactly distinct from one another yet having different arrangements, the body or the bodies do not appear as a natural element in the society but they are part of an entangled political strategies and tactics. While the body is not submissive to these technologies, it becomes possible within this material-discursive field that produces regulatory and corrective mechanisms for the life.

The regulated formation of social body, according to Foucault (1978), is not emerged within the level of ideas, words or speech acts. The situation is quite the reverse. That is, bodies are materialized in the form of "concrete arrangements" (p. 140) and they are made up within the entangled relationships of discursive practices, where the deployment of mathematics in its assembled form into the level of life is one of the technologies. The mathematically able body I am talking about, therefore, is not determined as an object and target of the power or it is not a linguistic construction that I am making up. It is real in a sense that we can only understand its materiality within the entangled relationships of the discursive assemblage of school mathematics. As Butler (1990) argues, the body has no ontological status without materialized acts, which constitute its reality, but the bodies are possible as a "result of a diffuse and active structuring of the social field [in which] this signifying practice effects a social space for and of the body within certain regulatory grids of intelligibility" (p. 178). That is, instead of assuming the subject pre-existing before the discursive assemblage of school mathematics, the mathematically able bodies suggest a historical subject constituted while doing school mathematics as an ongoing discursive practice.

These discursive practices, in this sense, are not purely language but ought to be understood "as places of what is being said and done, rules imposed and reasons given, the planned and the taken for granted meet and interconnect" (Foucault, 1984, p. 75). We can think the discursive practices as a form of rhizomatic map (Deleuze & Guattari, 1987), which is open to constant modification or any kind of social transformation. This map has to with performativities rather than so-called fixed competencies. Barad (2007) explains the focus of performativity in terms of matters of practices, doings and actions rather than turning everything into words and language. Then, mathematically able body should not be understood as a stable identity or a fixed body but it is performative, in which it has no ontological status without materialized acts constituting its reality, in the fluid webs of the discursive assemblage of school mathematics. Sociocultural variables or nonvisible mathematical concepts are not the only factors that constitute the reality of mathematically able bodies. The institutionalized discipline and control mechanisms that organize the pedagogical space and the society make the technologies on and of the body both productive and subjected to the knowledge. For example, these mechanisms generate "norms" for inclusion and exclusion ordering and regulating how to live and act in particular ways. The discursive assemblage of school mathematics produces "agential cuts" (Barad, 2007, p. 148) affecting itself and marking the other. That is, in Barad's (2007) agential realist account, the matter (i.e. school mathematics, pedagogical devices) is always an enactment and generates dynamic configurations within the discursive practices. The dynamicity of the performative discursive practices prevents the separation of material and discursive actors in the assemblage. None of them precedes one another; only the entangled relations can give school mathematics a meaning or some sense of existence.

3. "Math for All": Processes of Fabrication and Abjection

Mathematics education reforms are typically made under the phrase of "math for all" (i.e. NCTM, 1944; NCTM, 2000; NGA & CCSSO, 2010). While this proposition seems to be inclusive, the work begins with those who are "somehow" excluded. The phrase "all children" functions as a pivoting point to distinguish two human kinds in the standards and research: those who have all the capabilities to learn math and their disadvantaged others (Popkewitz, 2004, p. 23). "Math for all" embodies a process of including while simultaneously excluding by creating those who are yet to be integrated. While the discursive assemblage of mathematics education fabricates kinds of people (i.e. mathematically able bodies) through generating cultural theses, it simultaneously makes up the others. Then, it is necessary to think inclusion/exclusion not as a distinct process but intra-actively depending one another. The inclusion efforts can only be understood through the tactics and technologies of exclusion. In this sense, what makes mathematically able bodies are related to the making of "others". I follow Barad (2007) who problematizes the binary construction of the self and other while blurring the line between "subject" and "object":

> 'Able-bodiedness' is not a natural state of being but a specific form of embodiment that is co-constituted through the boundary-making practices that distinguish "able-bodied" from "disabled". Focusing on the nature of being able-bodied is to live with/in and as part of the phenomenon that includes that cut and what is excludes, and therefore, that what is excluded is never really other, not in an absolute sense, and that in an important sense, then, being able-bodied means being in a prosthetic relationship with the 'disabled'. How different ethics looks from the vantage point of constitutive entanglements. What would it mean to acknowledge that the 'able-bodied' depend on the 'disabled' for their very existence (p. 158)?

While the discursive assemblage of school mathematics produces a regulatory space for mathematically able bodies, as argued before, it simultaneously generates axes of differentiation. This delegitimization of the unlivable spaces is the making of abjection, yet these processes of abjection are also constitutive of the fabricated subjectivities (Butler, 1990).

Noting the intra-relatedness of self and other, the identities and identifications are not arbitrary incidents or psychic fantasies, but the concrete results of differential material-discursive enactments and the strategic position of knowledge as a material practice. These differential enactments that make self and other simultaneously are the interests of this study. The continuous embodiment of hopes for progress and development in mathematics education discursive practices and fears of human kinds who might be dangerous for that desired future, the "double gesture" of schooling, is making up mathematically able bodies while simultaneously invent their others. Processes of abjection and fabrication, then, can be considered as a way of thinking to historicize the “difference” in the discursive assemblage of school mathematics that make mathematically able bodies and their others possible as an object of research and teaching (Popkewitz, 2008).

Historicizing the discursive assemblage of school mathematics and problematizing this comparative style of reasoning challenges the taken granted notions such as mathematics itself. The principles of fabrication and abjection are to be reconsidered as a material-discursive practices of school mathematics that orders, classifies, differentiates kinds of people. Making visible the historical and political conditions along with scientific ideas that make “mathematically able bodies” object of governance in the discursive assemblage of school mathematics potentially opens up the alternative possibilities and unthinkable ways of living and being without dictating what they are ought to be.

4. Mathematically Able Bodies in the Political

While I am proposing “mathematically able bodies” as an object of governance in the discursive assemblage of school mathematics, none of the reform, policy or curricular materials or research practices on school mathematics have mentioned such a form. Mathematically able bodies are elaborated as an analytical tool to be studied in this research. This body is considered as a spatiotemporal configuration produced in the materiality of school-mathematics through educating children in mundane details of life. That is, there is a need to think mathematically able bodies as a historical construct that are moving from different layers in a manner that one builds and relates the one before; but extends and develops so there keeps transforming itself but with new technologies and tactics so they could be explained in a spiral set of connections.

Starting from the turn of 20^{th} century, we start to see emergence of new governmental rationalities and new configurations of productive power, which are qualitatively distinct, but not exclusive from disciplinary forms power that normalize the body as being, as Foucault (1997) notes. The transmogrified techniques and tactics, such as security and control, are becoming more and more apparent that take the life and biological processes of humans as species as a target, which is to be regulated if not normalized. While this seems like the older methods to administer the bodily enactments without targeting the individuals in the former societies of sovereignty, the necessary transmutations and modifications are made (Deleuze, 1992). Considering the question of population in the contemporary modern societies, Foucault (1997) introduces the biopower, which is quite different from the brute force.

> Beneath that great absolute power, beneath the dramatic and somber absolute power that was the power of sovereignty, and which consisted in the power to take life, we now have the emergence, with this technology of biopower, of this technology of power over "the" population as such, over men insofar as they are living beings. It is continuous, scientific, and it is the power to make live. Sovereignty took life and let live. And now we have the emergence of a power that I would call the power of regularization, and it, in contrast, consists in making live and letting die (p. 247).

The corporeal regulations generated within the discursive assemblage of school mathematics are governed by a kind of productive power that are similar to those can be called as "biopower" making the live of mathematically able bodies. In this sense, what we have is the positive regulatory function of school mathematics that takes the life of children as a political object to be governed through continuous and scientific management of mathematically able bodies.

In this configuration of contemporary modern societies, while different forms of power start to operate in new domains, there are still axes of differentiation and normalizations, which should be understood in correlative mechanisms rather than cause effect relations. That is, we have fabrication of kinds of people that is getting closer to the normal but not explicitly appropriating oneself to the norms. The normalization occurs, according to Foucault (2007), in establishing interplay between these different distributions of normal, not referencing oneself to the one specific norm. These distributions would serve as the norm and simultaneously marks others far away or deviated from the line normal. So, we do not have successive periods something like first disciplinary and then control in modern societies at the epistemic level, but the alterations in the transmogrification of the political, economic, institutional regime of the production of truth. What is brought into question in this study is onto-epistemological framework of discursive assemblage of school-mathematics that redirects attentions on the systems of reason that elaborates the ways of participation and action of subjects in their own local histories.

The ways of thinking in most of the mathematics education research embodies a particular notion of power that is inherent to social actors. The change usually starts with the identification of who has more power and then distribution of that power to other subjects in the playing field. The strategy is usually identifying the social, cultural and linguistic representations of mathematical identities, which requires the stabilization of self, body and mathematics. The critical genealogy of this field's legitimizing practices pushes me not to identify individual subjects of change but to rethink the entangled relationships occurred in the discursive assemblage of school mathematics that make subjects possible. Thinking these researchers as "conceptual personae", in

Deleluze and Guattari's (1991) terms, their solutions and efforts of change are only meaningful in particular time and place where the entangled relationships revealed from the discursive assemblage can give meaning to their words, not their individual biographies or not their personal ambitions about mathematics education practices. My focus, with a different style of inquiry, is to analyze how epistemological embodiments that make this playing field possible. To examine how discursive practices of school mathematics and bodies dynamically constitute one another requires going beyond the notions of culturally inscribed body; at the same time, opens up unthinkable possibilities of being and acting. That is, what I understand as a change.

In brief, this is a study of principles of "reason" that make mathematically able bodies and the categories of differences possible where "central to inquiry is the constitutive role of knowledge in the construction of social life" (Popkewitz & Brennan, 1997, p. 293). It also might be framed as disrupting the consensual practices and the normalizing effects of reason and knowledge. Yet this is not digging into the origins to see where this notion emerged, rather exploring around the assemblage of discursive practices to see how they are historically come together in an intelligible way to produce pre-emptive realities for the modes of living. The political critique requires the diagnosis and examination of the mechanisms that make things possible and seemingly held together. Historicizing is a way to deconstruct the long-held understandings and commonsensical practices of school mathematics.

5. Methodological Approaches

Mathematically Able Bodies as an Event

This study explores the historical and political conditions, material practices, along with the development of scientific ideas about teaching and learning school mathematics that make able bodies possible at two moments in the history. As mentioned before, mathematically able body is considered as a spatiotemporal configuration. I consider mathematically able bodies not as a historical constant but as a singular event, which can be analyzed in the within the assemblage of mathematics education discursive practices through its complex entanglement with a multiplicity of historical processes (Foucault, 1984). What makes these bodies possible requires a careful scrutiny of the analytics of power at the molecular level, which are very tied to the very depths of the society, following an analytical spiral. That is, mathematically able bodies exist in its singular configurations whose intelligibilities could be rediscovered again and again by showing the contingency of the arrangements in its discursive assemblage and showing the role of thought that make them hold together only in temporalities and in their localities. Then, mathematically able bodies are real in a sense that does not imply an ontological status of thingness but their realities can be understood through the agential relations produced in the mathematics education discursive assemblages, which does not subscribe a notion of truth based on their correctness (Barad, 2007). Nonetheless, how mathematically able bodies become possible as an object of governance in this discursive assemblage is to be questioned in this project through the critical genealogy of its own legitimizing practices. In this dissertation, the focus is on two particular moments of mathematics education separated by seventy years advocating "math for all" through a core curriculum. My purpose is to explore how the discursive assemblage of school mathematics produces particular kind of mathematically able bodies and their others, how their differences are made an object of intervention then and now. In this sense, mathematically able bodies are regarded as a historical event where it does ensemble the social and historical conditions, is produced through embodiment of new governmental rationalities and technologies and produces temporal subjectivities to maintain power relations.

Historicizing Mathematically Able Bodies

The method of this study is called as *history of the present* that is not about the past, but how "the past is intricately woven in constituting the present" (Popkewitz, 2013, p.440) in the form of grid. This grid enables to see how various discursive practices come together while the assemblage has its own intelligibility, which makes the conduct of people possible and "reasonable". The aim of this kind of historical investigations, as Rose (1999) mentions, is "to disturb that which forms the very groundwork of our present, to make the given once more strange and to cause us to wonder at how it come to appear so natural" (p. 58).

This dissertation is a historical study of the epistemologies, how we know what we know, of discursive assemblage of school mathematics. The analysis historicizes the two moments in the mathematics education (contemporary and pre-post war society, 1930s-45s) in order to make visible the parallels and discontinuities in these "math for all" projects that make particular kinds of mathematically able body possible. In both moments, the discursive assemblage of school mathematics employs various scientific practices and sociopolitical rationales to order and actualize the "difference" and functions regulatory mechanisms for everyday life. Historicizing the "reason" is a strategy to question the commonsensical practices of school mathematics in the name of progress and development. I am using epistemology or the "reason" in two ways. First, I analyze the reasons and historical conditions that make school-mathematics important in commonsensical ways signifying the social and scientific progress, thinking the discursive assemblage of school mathematics is a product of culture. Second, I look at the reason as hopes and fears of mathematics education that fabricate particular mathematically able bodies while abject their others. That is, the school mathematics produces culture. The comparison of two particular moments in the history is not to show accumulation of knowledge but to make visible the changing technologies and tactics to secure the power relations. Then, mathematically able bodies are not essential in character. It is a style of inquiry in this study to explore the shifts in discursive and non-discursive practices that make possible subjects of life, objects of research and differences to be acted upon. The question is, therefore, what material practices, historical and political conditions came into being along with new scientific ideas of teaching and learning mathematics and made the making of mathematically able bodies possible.

The question here is concerned with the reworking on the concepts that is given as natural or sacred in mathematics education such as mathematics itself. The onto-epistemological regime is brought into question through historicizing the legitimizing practices of the discursive assemblage of school mathematics. These questions would require rethinking current issues such as the inclusion and exclusion mechanisms in mathematics education when knowledge is considered in productive terms actively involved in the making of the child and the society. What kind of child is being produced? How is the child being ordered or develop their own reasons to live and act in particular ways? What kind of *un*livable spaces are generated? Who has become the abject in these discursive formations? What particular kind of child is fabricated through these formations? How does it become possible for the discursive assemblage of school mathematics as a productive agent to order, classify, normalize and differentiate the kinds of people?

It should be noted that this historical analysis denies the linear progression of time and it is not evolutionary. On the contrary, how we come to know what we know, epistemologies, are analyzed to understand how mathematically able bodies are constituted and how (fictitious) categories are formed.

This requires a particular notion of history and archival documents that legitimize its truths. In the sense of assemblage thinking, archive of this study is not limited with curricular documents on school-mathematics but includes materials from a variety of literature such as social histories on American culture, sources related to Enlightenment and histories of science that connects with mathematizations and some of the literature on contemporary reconfigurations of societies and identity politics. All of these collectively constitute the *archive* of this study.

I analyze the available sources not as an accumulated record of people's actions or intentions or as a story of particular institutions, but with a sensibility of the *epistemic habits* employed in these discursive practices that produces the ontologies for children, teachers and society (Stoler, 2009). The question is not why or what these statements indicating, but more of a how. That is, how we come to know what we know, the epistemologies in the constitution of ontologies are the unit of analysis of this study. Analyzing the epistemic habits enables me think about how the discursive practices of school mathematics make mathematically able bodies, the categories of difference as a site of intervention and what kind of pre-emptive realities are produced for kinds of people and the world.

The analytics of power relations, as described above, requires a detailed observation at the infinitesimal level while being politically aware of these small things, including but not limited to a whole set of techniques, a whole corpus of methods and knowledge, descriptions, plans and data (Foucault, 1975, p. 141). The complexity and the indeterminancy of these discursive practices would ultimately enable a molecular analysis of the epistemic uneasiness that creates the ontological formations *of* and *for* the bodies. Hence, I look at the keywords as indices of relations of power (Stoler, 2009, p. 33) through the cuts produced by the mathematics education discursive practices. When a keyword is kept appearing again and again (i.e. accuracy) in the making kinds of people and the world, the statements are in need of question by assembling with various discursive-material practices that make them possible. That is, what becomes the commonsense of school mathematics, teaching and learning assembled with the sociopolitical conditions of the society and the pedagogical and curricular technologies that make the able bodies possible is historicized to exhibit how the legitimizing practices for and of the bodies are the effects of contingent historical forces and political conditions.

Chapter III

Destabilizing Mathematics: From Discovering the Mathematical World to the Mathematical Modeling of the World

1. Introduction: Making the Mathematical World and the Self

The analysis of two historical moments of school mathematics, separated by nearly seventy years, reveals a continuous theme of making the mathematical world and the self. While mathematical investigations, possessions, representations or modeling seem similar and look unlikely to change, fail, or decline, these practices embody different arrangements depending on historical conditions that make them possible. Whether mathematics is in the world and is experienced by humans or mathematics is fabricated by the human mind, the notion of change in mathematics education reflects a tension between two different epistemological trends in mathematics. The debates about these epistemological positions constitute a division between realist and antirealist definitions for mathematics: Discovering the mathematical world or mathematical modeling of the world. While the former takes mathematics as something embedded in nature independent from humans, the latter does not assume that the world is inherently mathematical; humans can construct mathematics to model the world. Rather than moving between these two camps, which suggests a stabilized category of mathematics, my concern is about the "truth"; that is, how different ways of reasoning (about mathematics) announces new ways of finding the truth, introduces new types of objects and produces subjectivities (Hacking, 2002).

These two historical moments express crucial parallels in the discursive assemblage of school mathematics. The desire to produce knowledge, which is valid across time and space, remains prevalent through the notions such as "mathematical precision", "accuracy", and "validity of numerical operations". When the world is represented through mathematics, particular forms of subjectivities are constructed: The mathematically able bodies. These particular kinds of people, who are "able" to be true to the world as a contributing member of the society, are equipped with "mathematical power" that enables them to get closer to the "reality". On the other hand, the mathematical representations have their limits and dangers especially considering the distance produced between self and world in the process of making the self and world.

We should note that mathematics or numerical operations do not work alone as mere signifiers of particular categories or merely representing objective knowledge. On the contrary, they act through assembling with multiple discursive and non-discursive practices, institutions, cultural narratives, stories and particular logics about time and space. This assemblage of multiple practices speaks of differential constitution of subjects, how to know it and how to act on it.

This chapter is also about the "reasons" that make the shifts from "discovering the mathematical world" to "mathematical modeling of the world" possible. Having said the important parallels, to some extent the continuities, in the discursive assemblage of school mathematics, they are not exactly the same. They are not universal necessities regardless of time and space. On the contrary, there are multiple historical conditions, cultural logics and mechanisms of science that make these continuous ideas possible as well as their shifts. The consideration of broader historical transformations such as shifts to rational decision, anticipatory logics and the modifications in the calculation of risk enable to understand the new practices to find the truth, objects of inquiry and subjects of change.

I organize the rest of the chapter with respect to these two historical moments. Borrowing "the idea that what counts as mathematics is the product of a contingent history of human endeavors and the emergence of disciplinary boundaries" from Hacking (2012, p. 265), my aim is to problematize the binary construction of realist and antirealist accounts and question the category of mathematics circulating in school mathematics. Assembling with the gaze of pedagogical knowledge that regulates the social and pedagogical space that make self and society possible, my aim is to get out of the "unproductive separation between epistemology and ontology" (Popkewitz, 2008, p. 148) and to destabilize the "divine" category of mathematics circulating across the discursive assemblage of school mathematics.

2. Discovering the Mathematical World

One of the things prevalent in the discursive assemblage of school mathematics during pre-post WWII period was to think of mathematics as something observable in the world. While this way of thinking mathematics provided one of the "reasons" to legitimize the subject that was worth studying in the schools, it was, at the same time, a political anxiety. Studying mathematics in schools was to configure a technological and scientific culture for the society (NCTM, 1944). While the hope was to maintain progress and development in peaceful ways, the representation of the mathematical structures in nature generated a distance between the world and the self. This divide produced multiple subjectivities and their distinctions when the question was the reaching out of some form of truth, whether the representations were accurate or not. In line with these thoughts, in what follows my aim is to understand these emergent practices (in school curriculum) as a set of events and historical processes that make them possible.

2.1. Mathematics Education in the Emergency of War

Audible reasons why mathematics should be part of the required curriculum were social and scientific progress and development of the nation during the emergency of war. Beneath the surface, nonetheless, school mathematics was becoming an integral part of making of able bodies efficiently adjustable to American life and society through cultivating inner qualities of child. These reasons conjoined with the novel theories of teaching and learning mathematics that was directly concerned with the development of the ability to think in quantitative situations. Until this moment, it had not always been the case for school mathematics to facilitate the generation of the kinds of people for modern life. The crises and turbulences were resolved; the idea of "mathematics for all" had become intelligible for people through concrete arrangements. Mathematics, then, became possible to be referred as "mirror for civilization" and it was a competence that needed to be acquired by the youth to maintain the modern life. Nonetheless, the hope for civilized culture and democratic nation authorized the fear of "savages", "primitive societies" and "dictatorships". The double gesture distinguished two kinds in these reform calls, policy documents, general reports and scientific practices, the mathematically able bodies whose acts were regulated with this assemblage and their others who had to be rescued, saved or intervened. In chapter 5, my plan is to attend the issues of democracy, citizenship and the "all" in detail in relation to the bodily needs and interests. For now, my focus is to re-write a history for the category of mathematics itself.

2.2. Meeting with the "Power" of Mathematics

Mathematics, in the war years, was considered to bring a "scientific and technological culture" if human beings study mathematical structures of the natural objects and patterns in the world (NCTM, 1944). It has usually been as a taken granted subject matter that could be found somewhere in the nature waiting to be discovered. The ontological existence given to mathematics, as something to be discovered in the natural world, generated the "reasons" to study mathematics in schools as well. For example, in one of the calls that made urgent the need to solve the crisis in mathematics education deliberately suggested that:

> The material world, which surrounds us, reveals form as clearly as it suggests number. Although natural objects present themselves in countless varieties, certain forms have a tendency to reappear, and this constant recurrence suggests basic concepts and stimulates the creation of an appropriate vocabulary (Reeve, 1940, p. 3).

Mathematics is treated as passive, muted and distant subject matter existing in the material world in which humans are stimulated by their life sustaining needs to find and name it. The task of man was not only to "create an appropriate vocabulary" for understanding of natural objects but also to "struggle to survive" or to "make living more comfortable" where the nature is forced upon them (Reeve, 1940, p. 3). Solving the mathematics of the natural world would bring not only "scientific" understanding of the nature but would provide a condition to survive and make the life. The world was presumed to be already in a mathematical order. The creative part is to invent the tools, technologies and vocabularies to describe it.

We should note that "discovering the mathematical world" does not imply the superiority of the nature over humans. The mathematical power was not an innocent technology to interact with the world. It was making kinds of people and particular forms of culture:

> Men of culture in all lands have for centuries rated high a control of algebra as the universal language, a concept of the nature of proof as a guide to sound thinking, and a knowledge of trigonometry as a practical tool for problem solving (NCTM, 1944, p. 231)

Discovering mathematical world historically has not been simply admiring the mathematical beauty of the nature. The situation is quite different. Since every phenomenon in the world is measurable and countable, it is the Kantian subject who grasps his sense of superior self-worth through his awareness and reason over the nature. Nonetheless, the experience with the nature has a profound impact on the lives of individuals, the construction of American culture and the nation's ideas about itself, a sense of belonging to the republic as Americans. It is the redemptive task of "American technological sublime", Nye (1996) elaborates, which emerges in the conjunction of natural powers of Grand Canyon or Niagara Falls and the man-made technologies including railroads, skyscrapers, bridges, to stimulate a shared sense that can hold people together in the narratives of progress and development. Similarly, it was the "mathematical power" that would provide the ability of understanding the world and applying these knowledge for new and varied situations (NCTM, 1945, p. 209).

The "men of culture" who have a control of some sort of mathematical knowledge are not only concerned about the progress and development of the nation but also making particular kinds of people. While this culture demands the need for mathematical power in everyday life as a practical tool, it simultaneously intensifies the fears of degeneration or decay of that desired culture. Then, these hopes and fears generate and target those "uncultured" lives since they would deteriorate the unity of the nation. The task of school mathematics is to cultivate the characteristics of the mathematically able bodies and change them in a way that there would be no danger to the disruption of the social cohesion: "The schools must therefore aim to develop persons capable of unbiased and logical thinking and at the same time to mold character that will lessen the danger of unsocial employment of the power thus stimulated and strengthened" (Reeve, 1940, p. 30).

2.3. Mathematics across Time and Space

Constituted in school as mathematical, the ontological primacy of mathematics in the world suggests a mechanical, Newtonian view of world where space is distinct from body and time passes uniformly independent from anything that happens in the world. That is, while we discover more mathematics inherent in the world day by day, the cumulative accumulation of these discoveries are to be applicable basically everywhere in the universe. Douglas (1943) clearly mentions that the task of school mathematics was to find a "universal" knowledge through measurement and computation.

> This is a mathematical world. Everything we do seems related, in some way, to measurement. For a thousand years the world has been growing more and more mathematical. For a thousand years measurement and computation has been playing an increasingly significant part in the lives of everyone. The tendency to become at once the creatures and the masters of precise quantitative thinking and action is a rapidly accelerating phenomenon (p. 212).

The emphasis on the precision of the measurement practices to understand the phenomena under investigation is noteworthy. This suggests two things: First, mathematical reality is a natural given that is to be measured before the constitution of the subject. Put differently, whether humans interact or not, the world is already and absolutely in mathematical order. Second, the "creatures" and the "masters" of quantitative thinking are able to be true to nature and they get closer to that objective reality of nature through precise measurements. In effect, this produces a necessary distance between subjects and nature. Then, as I shall be arguing soon, the quantification processes are particular forms of interacting with the nature and the world in representation of the reality in objective ways. The more close the measurement gets to the reality, the more accurate the measurement is, which suggest the convincing character of numbers and mathematics rather than rigor or creativity. The objective representation of the space is to separate the self and the world. It is a technology of detachment.

> Man, through his age-long study of natural phenomena, has discovered the world to be orderly, predictable, knowable. With the use of an adequate number system, methods of precise measurement, and controlled experiment he has progressively discovered more and more relationships significant to him. The use of these discoveries produced the "industrial revolution," a scientific and technological culture (NCTM, 1944, p. 227).

This text suggests that mathematics was the method of discovering the world: The simplification of the natural phenomena into a controlled reality through the "use of an adequate number system", "precise measurement" or "controlled experiment". These mathematical methods make the phenomenon more seeable and more suitable to careful measurement and calculation that make possible the control and manipulation. Looking closely, these mathematical practices do not only entail discovering the world, but also make particular kinds of people who are using these methods as they try to understand the "mathematical world" and to discover the relationships that are significant to him through quantitative measurement and computation.

Mathematics was considered a "powerful" subject that could be utilized while interacting with the world. While the emergency and the uncertainty of the war construed mathematics as something promising in people's lives, the invention of quantification (which goes back to 18th century) also constituted discursive lines of school mathematics. Thinking in this way, I want to raise a set of questions rather than accepting tools and methods of quantification as they are. How can we think about these techniques and methods as a historical possibility rather than accepting as they are? What makes numbers, quantification or tools for measurement as appealing methods for discovering the world?

2.4. Quantification as a "Secular" Invention

One way to think of quantification and mathematical representation of the world is to situate these discourses around the Enlightenment, a European intellectual movement of the late 17th and 18th centuries emphasizing reason and individual, which accompanied the separation of church and state. Science emerges as the method of Enlightenment where individuals empirically observe the world in order to see oneself as an agent for action and change (Popkewitz, 2008).

If we think of the Enlightenment as a set of events and complex historical processes, as Foucault (1984) does, we can see mathematical tools and technologies to discover the world as part of this process. Quantification constitutes an important part of this emergent scientific method by providing a common language to standardize and locate the self in the universal knowledge and time. It joins with the secular republican American narratives (Popkewitz, 2008). The widespread desire for science and rationality, as opposed to the darkness of feudal church, make the production of objective knowledge that is valid across time and space brought the notions "mathematical precision", "accuracy", and "validity of numerical operations" to represent the world in particular ways and to give coherence for actions and participation in the world.

"Mapping mathematics onto world is always difficult and problematical," says Porter (1995, p. 5). It is a form of detached knowledge from the subjects and might miss important deep issues in the distance produced between people and the world, nature, social or any phenomena to which mathematics is being applied. While there are some limitations and dangers of quantification, the calculating agency of mathematical techniques, at that period, were considered a technology for managing nature and space and as a control of the man's environment. In fact, these techniques do not even discover the mathematical world, but make the mathematical world because the world is already so chaotic and disorderly. They do invent the organized nature and make the space calculable. At the same time, these quantification processes require particular subjectivities; that is, kinds of people. In this sense, considering both of the dimensions, "scientific forestry" can be exemplar for the abstractive and utilitarian logic to designate the space as applied to reason of state and culture of quantification (Scott, 1998).

2.4.1. Scientific Forestry: Control and Manipulation of the Space

Social and political transformations in Europe around the early eighteenth century yielded to embody the scientific principles for the administration of state, resulting in the quantification and rationalization of space as applied to nature and regulation of economic practice. During this period, as Lowood (1990) argues, scientific forestry management emerged as one of the largest sector of the state economy in central Europe since the forests were an important resource for the state economy and relatedly a vital necessity for maintaining the life of its people. In this emerging field, mathematics was a required subject for the training officials in order to describe the living forest quantitatively so that they could better manage, control and manipulate the disorderly nature. Nonetheless, in these processes, the goal was to demonstrate how the forester should proceed mathematically rather than producing new mathematics (Lowood, 1990, p. 322): Forest scientists were using numerical tables representing measurements and calculations and gathering data from the trees under specified conditions to determine, predict and control the wood mass to be used in the prediction of income, calculating the taxes, assessing the value of the forest and determine the damage of a potential natural disaster. The German forest, then, became an archetype for the neat arrangement of science to govern the disorderly nature through technologies of quantification and calculation (p. 340).

Governing, nonetheless, meant more than organizing the nature for the state economy. The mathematical practices, such as precise measurement, quantification, and generalization, that were mapped for the rational forestry were constructing the principles for scientific inquiry in particular ways. For example, as Lowood (1990, p. 331) mentions, the forester was instructing his assistants to correspond the mental picture of a tree to the numbers in the table so that they would become "a good forester" who were able to make instant associations between these two entities through sufficient repetition. That is, "the standard forester" was to create the standard tree, which could be realized through *learning to see* the typical tree not by studying and counting all the trees in the forest but a representative of them. Then, the scientific forester did not need to cover the every acre of the chaotic forest, what they needed to do was to sample and generalize, to read the tables in their office to make predictions and plans. It was not "direct real measurement" but the generalizations from the quantities determined mathematically, which formed the science of administration in Germany (p. 332).

German foresters were not only designating trees in order, but also generating the modes of conducting a "scientific" inquiry, which were the calculation techniques employed to render the process more objective and reliable. As Porter (1995) would argue, "strict quantification, through measurement, counting, and calculation, is among the most credible strategies for rendering nature and society objective" (p. 74). So was the scientific forestry. It was the mathematics that could constrain the desires and biases of individuals as well as provide a *trustworthy* explanation for the world. Being true to nature, in fact requires not only methods to produce objective knowledge but also restrained processes where imagination and judgment were in suspect because they are unruly to the scientific discipline (Daston & Galison, 1992, p. 118). Then, trustworthy explanations embody a positive and objective image for the scientist and the scientific knowledge. The important point is for us to recognize the task of quantification is to find "a close fit between mathematics and a select set of phenomena", which is a form of "moral economy" of doing science (Daston, 1995, p. 8). Being faithful to original echoes Christian asceticism (p. 21), where scientists self-discipline their judgment, interpretation and aesthetic values in order to be accurate and precise. This reveals a particular formation of the character of measurers as moral agents and simultaneously the quality of measurements. Then, we can say that quantification was a secular invention and enacted by replacing God(s) with mathematical tools and technologies to calculate social and natural space and to govern people.

While the utilitarian mathematical simplification of the forest was to maximize the wood production in efficient ways in the short term, the identification of particular elements from an exceptionally complex nature, such as a forest, incorporated the danger of dismembering some of the materials, which seemed as unrelated. This, at the end, produced forests as monocultures (Scott, 1998). The standardized forestry became vulnerable to the contingency of unplanned natural events such as storms, drought, severe cold or condition of soils, which yielded to the invention of new sciences of intervention such as "forest hygiene" (Scott, 1998). Then, the task of the "scientific culture" was to identify those fragile trees and restoration of them in order to maintain the "mathematical" order of forestry. *So, quantification of space and the particular forms of knowledge generated from these processes had their own problems. In fact, these generalizations could be* *treated doubtful rather than trustworthy.* Underneath these practices, however, was not only to manage the nature but also to scratch the characteristics for particular kinds of people and to create the culture of doing "science" as moral agents through the use of mathematics as a tool to describe and generalize a social or natural phenomenon.

2.5. Civilizing Qualities of Mathematics

Civilization is one of the most mentioned legitimizing reasons to pursue the study of mathematics for the child and the youth. "Mathematics is mirror of civilization" (Reeve, 1940, p. 2), where it received contributions from different people and it was to provide a common heritage for human beings for social and scientific progress. While the idea of progress, considering broader cultural context of America, was a historical hope for a new moral and social order led by enlightened and virtuous men who were forward-looking with a desire of planning the future (Wood, 1991, pp. 189-190), it was, at the same time, a fear that how *primitive* the civilization would be without those mathematical virtues (Reeve, 1940, p. 5, my italics). The task of school mathematics was to cultivate the inner qualities of the child to become a moral member of the civilized society with accurate and clear thinking.

> Man sets ideals for the things he does, and accordingly an ideal must be set for thinking. If accuracy, cogency, should be this ideal, then it is attained in mathematics. That is why mathematicians are little troubled by the question whether they are engaged in the most human of enterprises, accurate thinking, and are not demanding immediate outcomes. Can man by reasoning arrive at conclusions that represent in highest conception of truth? The answer is yes, and it is the constant business of mathematics to show this (p. 9).

The conjecture was that mathematics would give a good measure of accuracy, cogency and rigor through embodiment of mathematical thinking and reasoning, what to say, how to explain it and how to represent it. The words "reasoned" and "mathematical" explanations do not refer only doing mathematics, but also making kinds of people as these provide the standards of explanations, justifications and making conclusions. Verbal reasoning, as Whewell (1831; as cited in Porter, 1995) argued, was too vague or imprecise to be tested against those uncompromising judges, experiment and observation. It was the mathematical explanations that could overcome the shortcomings of other forms of reasoning.

Civilization emerged as "a story of the evolution of universal humanity" through application of reason in the French Enlightenment. The universality was transcendent and outside the history and operated as mutual construction of self and other where "man" was scaled "in a continuum of value and hierarchy to order and divide people, races, and their civilizations" (Popkewitz, 2008, p. 35). The location of the self in this hierarchical continuum between savage and civilized was not only about the self but also society. It was to construct civilized nations under the siege of reason, science or mathematical technologies while racialization of others and legitimizing the intervention.

As conveyed in the justifications of school mathematics as a required subject in secondary education, to illustrate, the "Greek achievement" in geometry was the result of a rational science. This rationality was about providing a reasonable and mathematical explanation of natural phenomena and numerical processes generated those explanations, not the invention of the geometry as itself (Reeve, 1940, pp. 3-4). The initiation of rational science through these reasonable argumentations were claimed to serve the foundation of Western civilization and the truth embedded in the particular modes of doing scientific inquiry and in the processes. Nonetheless, in fact, these rational arguments were more of a technology of trust and moral economy given the "absence" of Gods in the Enlightenment. In short, this particular way of doing (school) mathematics was a communication mechanism operating as a consensual practice that would provide not only harmony within the groups but also stability of the social order and state economy.

In the era of European achievements, says Hacking (2012), Greeks were glorified as they "discovered new mathematical facts and structures"; nonetheless, from Kantian view, what counted most as revolutionary was the invention of proof. The ability to make demonstrative proofs was uncovered as an "innate capacity" of all human beings (p. 269). Then, the doing mathematics was not even about discovering mathematics in the world but the *ability* to demonstrate, prove and justify. When these long-held understandings ensembles with the principles of schooling, we shall see how these abilities are ordered developmentally in the next chapter. For now, bringing these together, it is important to note what counted as civilization was not the mathematics itself but the particular way of reasoning about the world. This style includes ability to make rational arguments, justifications and demonstrations to find the "truth" that has to do with the governing subjects (people engaging with mathematics) as moral agents and trusted beings.

Connected with the reason of schooling, "civilizing" qualities of mathematics, interestingly, transmuted into "creativity" in the efforts of legitimization of mathematics in the school curricula:

Creativeness may also be encouraged in discovering and formulating problems, in devising methods of attack, in recognizing relationships among data, in discovering methods of proof, and in presenting conclusions in expositional or other forms. But if mathematics is to be a field for creative activity, the approach to problems must involve a type of investigational experience which is an adventure into the unknown- it must provide constant opportunity for discovery (PEA, 1940, p. 52).

2.6. Publicizing Mathematics as a Mode of Civility

"Much of the modern work", reports National Council of Teachers of Mathematics, "involves precision and minute attention to details, such qualities becoming more pronounced as civilization grows more complex" (1940, p. 27). The mode of civility here embodies particular forms of reasoning about the reality in the making of self and world such as precision, accuracy and attention to details to reach the truth. Nonetheless, taking these arrangements of reason and rationality as signifiers of civility authorizes the fear of primitiveness, decay or darkness. Then, this mode of thought did not only incorporate its boundaries as a technology of distance between the self and the world, but also made up particular kinds of people and produced differences between different kinds.

Eighteenth century rationalism was the light directed against the fogginess of the feudal system and darkness of the church (Heilbron, 1990). The political philosophy of Enlightenment spread the notions of reason and rationalization realizable by numbers (Heilbron, pp. 22-23). It was the rational science that would secularize state against the church and would bring freedom, yet this had to be done within the boundaries of quantitative entities, which generated the standards of truth-making and stabilization of factual knowledge. For about two centuries, as Porter (1995) argues, quantitative precision was central to experimental rational science; nonetheless, the quest for precision was more to do with moral economy than theoretical rigor (p.

50). The precisely quantified entities were to provide trust, agreement and to maintain the moral order both in doing science and in the social relationships. Quantification, which was an indication of persistence, ability and objectivity, became possible as a crucial agency to govern the people. It was to identify *type of persons* who would not, could not, lie (Shapin, 1994, p. 410, italics original).

Moral economy of science is not about external forces but the practices of self-control when conducting scientific inquiry. They are "integral to scientific ways of knowing", says Daston (1995, p. 7), "[they] are historically created, modified, and destroyed; enforced by culture rather than nature and therefore both mutable and violable". They are not absolute rules that put pressure on the scientists. However, this does not mean that they do not have limits and dangers.

Considering mathematical investigations that seek "accurate" explanations for the world and desire to be "faithful" to the original, moral economy of doing objective science has started to regulate the conduct of scientists during nineteenth century. That is, the perpetual self-control of scientist and the employment a positivistic attitude are necessary for the "scientific sociability" across continents and oceans (p. 22). As Elias argued (quoted in Daston, 1995), the complex coordination of human activity was a form of civilization process and science was not an exception. The "civilizing" processes of conducting inquiries were constitutive for the collective culture of objective science. However, this particular culture honor particular objects of study at the expense of other, trust some forms of evidence while exclude other ways and configure character of the particular kinds of scientists (p. 23). The moral character of mathematics was a sign of honest and careful work, protecting science against false judgment and bias (Porter, 1995). Mathematical precision was the civility of scientific practice and the moral economy of those who testify about the nature and the world. Then, the person, who wanted to do science, was to equip himself with mathematical knowledge (Shapin, 1994).

As the natural meets with the rational and the universal knowledge, those scientific people could reach consensus guided by mathematical ways of knowing. However, this reason was not the property of a group of persons; the truth needed to be accessible to all people as they have the right to know and recognize the universally intelligible knowledge (Heilbron, 1990, p. 210). If Enlightenment was against the darkness and dictatorialness of the feudal church, mathematics was a practice for free people. Quantification was to make self-directed individual to make a rational and a secular society. Mathematics was a competency not only for the service of science, it was, at the same time, a discipline for all who "concerned to set culture on a more moral and rational footing" (Shapin, 1994, p. 320).

It was the task of school mathematics, in a similar vein, to create that "man of culture" with "a control of algebra as the *universal language*", "a concept of proof to guide *sound thinking*" and "a knowledge of trigonometry as a *practical tool* for problem solving" to deal systematically with the more general problems of society involving quantitative thinking (NCTM, 1944, p. 231, my italics). Then, school mathematics was more than teaching mathematics, but it was to make particular kinds of rational people as safeguards of the nation to maintain the moral order of the society through production of impersonal practical knowledge or truth. It was the re-inscription of the historical concern of governing people with numerical operations and mathematical precision in modern societies: "In a world in which the questions 'how much?', 'where?', and 'how many?', have to be answered again and again accurately and with great precision, mathematical literacy is almost as important as the ability to communicate" (NCTM, 1944, p. 227). Then, the materialization of the world as mathematical was a historical possibility rather than a sacred fact.

2.7. Some Final Remarks on Pre-Post WWII Reforms and Practices

In this part, I have scrutinized the discourses circulate in pre-post war period that make "mathematics" as a legitimate subject matter in the school curricula. The examination of these practices revealed a close relation with the Enlightenment's theses on reason and rationality to create a civilized culture for progress and development. In a similar sense, "the possession of mathematical knowledge" was the hope for creating a civilized culture and maintaining the peace considering the dark and uncertain atmosphere of the war. So, planning for the post-war period was imperative and mathematics education was part of that planning. However, these solutions and plans for action operated beyond teaching and learning mathematics for progress and development. Discursive assemblage of school mathematics produced governing mechanisms for people and categories that distinguish them.

In this context, the sequential track had to be organized for the children, the potential truth-tellers, who want to pursue science and technology related careers in order "to provide sound mathematical training for our future leaders of science, mathematics, and other learned field" (NCTM, 1945, p. 195). The rest, who constituted as the Others, had to placed in different tracks. I shall be arguing more about these differential technologies, tactics and what constituted different in the next chapters. However, it is important to note the following for now. The location of the self in this hierarchical order of tracking was not only about making the self but also making the society. The planning of sequential track was to plan the society for the post-war America in which technological culture would flourish in the scientifically oriented society in the peacetime. The scientific self and society was going to become the redemptive narrative for the nation following the catastrophic effects of the war. Those who possessed "mathematical power" would become the moral leaders who would correct the social wrongs and would rescue the masses and populations, which psychologist G. Stanley Hall once referred as "the great army of incapables".

3. Mathematical Modeling of the World

In the contemporary practices of the discursive assemblage of school mathematics, we start to see "mathematical modeling" as one of the standards for the mathematical practices in the most recent reform initiatives and policy practices in the United States (i.e. NGA & CCSSO, 2010). Mathematical modeling, as a standard of mathematical practice, is not a collection of isolated topics but should be considered in relation to other standards (p. 57) in the making of self and the world in its flow across school mathematics. Before getting into what mathematical modeling does, and what are its limits and dangers, let us look at how this emergent mathematical practice is articulated:

> Mathematical modeling is used to explain phenomena in the real world and/or make predictions about the future behavior of a system in the real world. Of equal importance with the fact that mathematical modeling starts in the real world is the reason why it starts at all. The process of mathematical modeling is intended to help the modeler understand or predict something about the real world and to develop theories and explanations that provide insight and understanding of the original real-world situation (Cirillo, et. al., 2016, p. 8).

The "reason" of school mathematics remains in the movement between the real and the mathematical that produces a gap between the self and the world similar to the pre-post WWII period. Although the term literally appears in the contemporary practices, the reasoning about the world through mathematical representations constituted the commonsense of the discursive assemblage of school mathematics in the past as well. Nevertheless, there are significant differences in the exercise of the contemporary discursive assemblage. Since the Second World War, there are considerable shifts in how people think about themselves, reason, rationality and the real world. Maybe more importantly, school reforms are extending beyond the national boundaries and operating in the "borderless globe". Mathematics education is also part of these transitions. In this part, I shall reconsider "mathematical modeling" as a product of different discursive practices and how it makes up particular kinds of people and the world.

3.1. School Mathematics in the Emergency of Securing Uncertain Futures

The proliferation of the "realistic" processes in the form of "mathematical modeling" is found in the global sphere. Mathematical modeling is "a cornerstone of the PISA framework for mathematics" where it "assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions" (OECD, 2013, p. 25). These international exams are not only concerned with the assessment of mathematical knowledge, but also with the ability to "formulate a situation mathematically" (p. 28), "translate that information into a useful mathematical form" (p. 29) and "predict a phenomena" (p. 25). "These cognitive capabilities", reports OECD, "are available to or learnable by individuals in order to *understand and engage with the world in a mathematical way*" (p. 30, my italics). Mathematical modeling becomes a particular salvation story that takes the world to be mathematically organized and facilitates the decision making in the uncertain situations and predictions about future.

The embodiment of these corporeal regulations (i.e. engaging with the world in mathematical ways) is not only making the self but also the world. The hope for preparing young people for life in modern society as citizens in the modern world, in fact, ensembles with the Enlightenment's hopes of the world citizen committed to ideal values about humanity and the fears of degeneration and decay (Popkewitz, 2008). For example, one of the assessment tasks from the PISA 2012 exam is about the graphical representation of the decomposition time for the different types of litter such as banana peel, chewing gum or polystyrene cups and (OECD, 2013, p. 51). This is an exemplary task in the "scientific context" of the individuals' lives where the aim is to measure mathematical capabilities as interpreting, applying and evaluating mathematical outcomes. Mathematizing of the "realistic" situations is to govern the moral conduct of the child through "engaging with the world in mathematical ways". They are to plan the societies and the world in creating and securing this moral order as "[the] assessment at age 15 provides an early indication of how individuals may respond in later life to the diverse array of situations they will encounter that involve mathematics" (p. 24). Nonetheless, this process is not only about governing the conduct of the child, but also functioning as technologies of self. What lies beneath these lines is a goal to make particular kind of person who is committed to science, humanity and world through taking care of nature, life of self and life of others.

The idea of making the "world citizen" prompted an "international benchmark movement" in education. While United States has largely ignored these movements initially (National Governors Association [NGA], 2008), calls to take action to "improve" educational practices were not postponed. "In a knowledge-based, globally competitive economy" says Duncan (2014), the secretary of the US Department of Education, "education is the new currency, and this currency is recognized internationally" (p. 25). He continues with the necessity of "problem solvers" by saying that "applied math and science skills of the kind measured by PISA are essential to propelling innovation and maintaining international competitiveness" (p. 26). The hope for mathematical problem solvers in the knowledge economies and the notion of competitiveness reinvent the salvation themes of American narrative: How can students from Ireland, Poland, Latvia, the United Kingdom and the Czech Republic outperform in mathematics than their US peers (p. 24)? The collective investment in human capital and the hope for mathematical problem solvers in the knowledge economies are articulated as a form of defense in the "borderless globe" (NGA, 2008):

> We are living in a world without borders. To meet the realities of the 21st century global economy and maintain America's competitive edge into the future, we need students who are prepared to compete not only with their American peers, but with students from all across the globe for the jobs of tomorrow (p. 1).

We can think of these lines as the reconfiguration of the American race and its prosperity across the world. We should additionally think and understand how hopes and fears of our contemporary times operate differently. There are predictions that demographers make about the changing population in the United States. When half of the high school graduates are "so poorly prepared" today, America is not able to "thrive in the global knowledge economy". The implication is that "states that plan to grow their economies *must* find ways to close their achievement gaps" (NGA, 2008, p. 14, italics original). Then, the fear is not the past or the present, but the uncertain future. When this demographic shift happens, America should be ready for the changes. This has to do with regulation of the present.

3.2. Tensions and Movements in the Mathematics Education Community

While mathematical modeling might seem to appear in a sudden or a top-down way, mathematics education (research) community has already been tackling with this kind of practice for decades, especially around the standards movement that emerged around 1980s. The experience of "strong shifts back toward basic skills" and the growing recognition of "a serious mismatch" between curriculum materials and success beyond school triggered the mathematics education community to reconsider the nature of mathematical activity and to make the pendulum swing back toward problem solving (Lesh & Zawojewski, 2007, p. 764). It was not until this time, mathematics education researchers change their thinking on "the nature of complex mathematical activity" (Lester & Kehle, 2003, p. 501) as they started to realize the growing distinction between "mathematician's mathematics" and "school mathematics based on skill" (Lesh & Doerr, 2003). Between 1930s-1970s, as Lester and Kehle (2003) mention, the cognitive processes were assumed to be essentially the same for all kinds of problems[1]. The transfer of knowledge between different contexts and the existence of a universal theory of problem solving came into question. Also, the expert-novice dichotomy became unintelligible since the same task could be considered as problem for the novice and as exercise for the expert. The focus on individual expert or novice problem solvers in the case studies provided little clue about the processes of mathematics activities. While cognitive scientists recognized these processes differ across knowledge domains and the findings may not necessarily be generalized to situations outside the laboratory, it was not until 1980s that mathematics education researchers began to call

[1] By "problem solving", authors refer to the problem solving practices rooted in the European Gestalt tradition, which is not necessarily connected to the real life issues and specific subject domain but the research done with simple laboratory tasks from psychological perspective focusing on individual performance. See Lester & Kehle (2003) for more information.

for integration problem solving with a domain-specific focus such as studying problem solving in the context of learning and doing mathematics, rather than as a separate topic (pp. 502-503). However, the issue that interested the mathematics education community was more than this. They wanted to know how mathematics learned in schools was viable in being able to work and live in dynamic environments outside of school. The integration could not only be limited to fields of study (i.e. science, technology) but also the real world. What needed were a "forward looking" and a "fresh perspective" for the changing science, technology and the world (Lesh & Zawojewski, 2007, p. 780) and one of these perspectives was "mathematical modeling".

Already employed as a practice in a variety of disciplines (i.e. biology, engineering, finance, computer science, and the social sciences), it is not difficult to answer the question that asks, "why (mathematical) modeling?" Mathematical modeling is seen as integral to the practice of science, as a vehicle to integrate traditionally isolated (school) subjects and as a way to interact with the natural and social world (Cirillo, et. al., 2016, pp. 13-14). However, the most crucial element in mathematical modeling is the process that links "mathematics" and "authentic real-world questions" through translation of a situation that is not inherently mathematical (Cirillo, et. al., 2016, p. 5). As Pollak (2003) writes in his review of the history of mathematical modeling in school mathematics, what makes mathematical modeling distinct from other forms of application of mathematics is the "*explicit* attention" to the "*process* of getting from the problem outside of mathematics to its mathematical formulation" and the "explicit reconciliation between mathematics and real world situation at the end" (p. 649, italics original), where he focuses on the "represent[ation] a real-world situation by a mathematical one" (p. 648). The process of mathematizing of a real world situation is considered as a form of mathematical thinking what

Lesh and Doerr (2003) call "conceptual systems" (p. 10). As humans develop conceptual systems, which include elements, relations, operations and interactions, to make sense of their experiences through mathematical modeling activities and they can also use these modeling devices or systems to predict and explain other complex systems (p. 15).

At this point, the mathematics education community has many "reasons" to engage with mathematical modeling as one of the mathematical practices. They seem to agree with the failure of knowledge transfer and to recognize the complexity of real life situations. Mathematical modeling is thought to have an "unrivaled success" (Cirillo, et. al., 2016, p. 8). Nevertheless, the move from "discovering the mathematical world" to "mathematical modeling of the world" is possible through the emergence of the new keywords such as process, system or prediction, which should be situated within the broader historical and political conditions along with scientific mechanisms. In the next part, I trace various discursive and non-discursive mechanisms since the Second World War to see how mathematical modeling becomes possible as an undoubted practice in mathematics education.

3.3. The Materialization Rational Choice in the Process

In the reflection o a catastrophic event, the Second World War, postwar researchers started to rethink the scientific theories about human rationality and they began to orient their work towards the science of decisions with organizational management theories (Heyck, 2015, p. 126). The limits of human reason and the consequences of those limits for democracy shaped the development of the sciences of choice. The problems and the crises of modern social thought (i.e. making the human rational and reasonable) shift the focus from decider to decision, from the person to process not in the abstract theoretical way but producing the decision in practice or in the process. There was an explosive interest in decision making to solve the dilemma of irrational behaviors of man and the hope to find a safe ground for stability in science and society (p. 130). The effects of this shift to reasonable being to reasonable choice were going to be experienced in the mathematics education through recognizing the failure of knowledge transfer and the complexity of the world. The once unquestioned necessity of "possession of [mathematical] knowledge" (NCTM, 1944, p. 227) is no longer promising while mathematical modeling, which is "the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve *decisions*" (NGA & CCSSO, 2010, p. 72, my italics), has started to seem more hopeful.

The emergence of this particular type of organization and a mode of thought invested in decision making is not a complete flight from shaping and fashioning individuals' thoughts and actions. The difference is only the reconfiguration of individuals as particular kinds of people with a reference to the system. It is not important who made the decision. In this emergent mode of thought circulating in the social sciences, the unit of analysis is the choice not the chooser. The point is to produce a "rational choice" by any kind of system and process (Heyck, 2015, pp. 132-134). In a similar sense, mathematical modeling that links the classroom mathematics with everyday life focuses on the decisions made in the process (NGA & CCSSO, 2010). Nonetheless, this requires a particular kind of person to make this process happen. The target is not only the decision or choice produced in the system, but also the bodies flexibly attached to the system to maintain the equilibrium.

We should note that decisions are not made in certain conditions. One needs to decide when there is uncertainty in the context of anticipating the future (Amoore, 2013). The decisions produced within a system include processes such as "scenario planning, risk profiling, algorithmic modeling, information integration, and data analysis". However, these processes of decision-making are becoming "the authoritative knowledge of choice" (p. 9) in the anticipation of the future through the mathematical models.

The focus on the process as chooser rather than a person reveals different relationship with time than the earlier reforms. The past, traumas or repressed fears in Freudian sense, is not important. The process should start from the present and decisions should put a light on future, it should be always forward looking, not back to the individual's past. This is not to say past was irrelevant, but impractical (Heyck, 2015, pp. 132-133). Likewise, mathematical modeling needs to start from the present, from the real world, in order to explain the current system and to make prediction in relation to it. Like Cirillo and her colleagues (2016) claim, "of equal importance with the fact that mathematical modeling starts in the real world is the reason why it starts at all" (p. 8).

3.4. Homologous Qualities of Mathematical Modeling

There is much written on mathematical modeling, and although the main idea is to link mathematical practices with authentic real world situations, there are various perspectives and distinct approaches on conceptualizing mathematical modeling (e.g. see Kaiser & Sriraman, 2006). Taking a different approach, however, I examine the undoubted elements of mathematical models across different studies. This is not to reduce them into sameness, but to see the homologous features across them. In short, the characteristics of mathematical models in the context of school mathematics converge into four main points. First, they are *predictive*. In connecting the real and mathematical, the aim is to forecast what comes next. Second, models are *explanatory*. They are not only for predicting something about the future, but tools to describe the situation and represent the world. Third, mathematical models are *useful*. They need to be practical in engaging the world and simple enough to reuse. Lastly, mathematical modeling is a *creative* process. The modeler needs be creative to mathematically engage with the world. In the remaining of this section, I unpack these converged elements of mathematical modeling in depth; try to understand what these characteristics do, what the limits and dangers are. Although I separate these elements, they do interact and influence one another. The point is to examine what makes modeling reasonable, what the paradoxes are in this grid of intelligibility and to unsettle what appears natural.

3.4.1. Mathematical Models are Predictive

Mathematical modeling does differ from the previous practices of school mathematics. One of the most noticeable features is their potential to provide an account for the future by focusing on the present situations. What temporalities might be produced when the focus is on the complexity of present situations to predict the future? Mathematical modeling does not assume the world is inherently mathematical, as in the earlier reforms. The starting point is not the discovery of mathematics existing somewhere but instead, to find an external reality to be mathematized. These (mathematical) models are providing some sort of futuristic perspective in relation to the ever-changing world and a new kind of knowledge as they have the potential to become more situated and more contextual. Models are not fixed, stable or structurally unchangeable, they are "manipulable mobiles" but in rule-governed ways (Heyck, 2015, p. 166). The dynamicity and manipulability of models provides a new kind of fluidity between image and thought, where movement comes into being in the form of mathematizing. This is, indeed, a process operationalized in the context of system, what Freudenthal (1968) calls "the process of mathematizing reality" (p. 7), which is frequently referenced to in today's mathematical modeling research. In explaining, "how mathematics can be useful", Freudenthal points out a dimension of time-space when individuals apply their theoretical knowledge to practical use (p. 4). While this application of knowledge in real life in the form of "mathematizing reality" is its infancy in the 1960s and particularly in Europe, it has become foundational in the Realistic Mathematics Education, a domain-specific instruction theory, where "rich, "realistic" situations are given a prominent position in the learning process" (van den Heuvel-Panhuizen & Drijvers, 2014, p. 521).

The futuristic perspective employed in mathematical modeling enunciates a "not-yet-realized aspiration to transform a world of ontology, description, and materiality to one of communication, prediction and virtuality" (Halpern, 2005, p. 287) that held by also cybernetic researchers in the Cold War period. As a science of control or prediction of future action, cybernetics was not only concerned about describing the world as it is but also to predict what the world would become through producing "a range of probabilistic scenarios". Unearthing the dynamic and multiple processes between thought and action produced a mode of thought, invested in prediction and concerned with the transmission of information to control the future (pp. 287-290). One could argue that mathematical models are only producing possibilities; however, there are politics of possibilities.

3.4.1.1. Big Foot Problem: Predicting and Identifying the Person of Interest

Let us take a widely used and referred to mathematical modeling activity: Big Foot Problem (i.e. Blum, 2011; DeMatteo & Johanning, 2010; Lesh & Doerr, 2003). For this model eliciting activity, students develop a mathematical model that allows police to find the person through looking at the footprints in the event scene. The aim is to produce a "'how-to' tool kit" that would help the police to make good guesses about how big person it is in relation to the footprint available in this particular case and can be usable for other footprints in the future cases (Lesh & Doerr, 2003). In order to make the problem more authentic, students are provided a newspaper article about a famous tracker who often works with the police to help them to find "lost people" or "escaped criminals" by looking at the footprints. This person is able to make "accurate estimates" about the suspects who made these footprints such as "how tall they are, how much they weigh, whether they are men or women, and how fast they are running or walking" (p. 5).

Students collect real data for this activity; they measure their own heights and shoe length to generate a conceptual system or a model that would allude to a proportional relationship between footprint and size. While the mathematical objective of this activity is to measure the lengths and to calculate the proportions, the relationships between numbers are producing new and actionable realities or "data derivatives", in Amoore's (2011) terms, through the inferences based on the correlations. For example, if the length of the foot is big, then the suspect is taller and most probably male. Seemingly abstract and value-free calculations make it possible to reach a form of knowledge to predict and identify person of interest.

It is important to note that the highest order of "interpretation" or "modeling cycle" does not necessarily involve the qualitative judgments about the size of footprints for different sexes or sizes. When students, for example, use only "qualitative judgments" such as "This guy's huge" or "You know any girls that big", this is regarded as the lowest order of interpretation or conceptual system (Lesh & Doerr, 2003, p. 19; Lesh & Harel, 2003, p. 165). On the other hand, the highest rank of the interpretation suggests that "students are being very explicit about footprint-to-height comparisons" whereby indicating a proportional relationship such as "a person's height is estimated to be about six times the size of the person's footprint" (Lesh & Doerr, 2003, p. 21).

These calculations do not only produce conceptual systems for teaching and learning mathematics, but also create ontology of associations based on the data. The inference or prediction is not made regarding the particular but on the basis of average person. That is, this mathematical model has to work for other footprint situations not in determinate ways but taking the uncertainties into account through the notion of variations and probabilities. These concepts including but not limited to average, variation or probability has historically become a way to control future through numerical calculations such as ratio, proportion or multiplication (Hacking, 1990). Nonetheless, while these numerical calculations intend to depoliticize real world situations by sanitizing qualitative judgments (i.e. gender, race, size), they simultaneously re-politize through generation of pre-emptions (Hansen, 2015) and ensemble with the qualitative information such as race or gender. Without these numerical calculations, one cannot act on a footprint. But with these associational ontologies generated by "mathematizing processes" or derivatives coming from the data, it becomes possible to act in the face of uncertainty, danger and risk. That is, future becomes a category to be acted upon in the present.

Considering the discursive assemblage of school mathematics, multiple preemptive realities are produced in this modeling activity. First, the data derivatives, the identifiable characteristics of the suspects, are generated through the numerical calculations. They create regulated capacities to act and participate in "real" life. Second, mathematically able bodies who have the will to take control of their life in the face of uncertainty become possible and to some extent reasonable. The epistemological anxieties operate in two spheres: The anxieties about securing the uncertain futures and the anxieties about making up people who willingly take care of their security. For instance, the features of highest ranked modeling cycle of the Big Foot Problem suggest to move beyond qualitative aspects of the situation and to make "precise" measurements so that create more "accurate" judgments, which in fact re-enunciates the moral character of the measurer. These notions of precisions and accuracy function as a technology of trust, in Porter's (1995) sense, and make these data derivatives intelligible. Maybe more importantly, these modeling cycles are ordered, ranked and associated with the psychological growth: "The modeling cycles that students go through often are strikingly similar to the stages of development that psychologist and educators have observed concerning the natural development of the relevant constructs" (Lesh & Doerr, 2003, p. 21). I will be extensively engaging with these developmental narratives in the next chapter, there is more to say but for now I am leaving it here.

3.4.1.2. Calculation of Uncertainty and Making Kinds

Mathematical modeling practices do not reveal strictly linear time and they are anticipatory. The emphasis on risk and decision-making reconfigures that "future uncertainty can be acted upon in the present, even when there is little or no knowledge of past instances" (Amoore, 2013, p. 62). The new relationship with the time, in fact, carries its own ironies. These probabilistic scenarios are about prediction of future behaviors of the each component of the system, including humans, but from the accumulated data of previous interactions. In the analysis of behavior-functional systems, what underlies the surface is the need to have a social theory and application of it to the social and natural phenomena (Heyck, 2015). This shares the experimental philosophers' view of the progress of knowledge and improvement of the world as in the Enlightenment. The application and production of mathematical models are to render society more objective and certain with rational decisions. In this text about mathematical modeling processes, we can see how chance is tamed in Hacking's (1990) sense as mathematical modeling involves "simplification of a complicated situation", "identifying important quantities in a practical situation" and "mapping these relationships using tools" such as "diagrams, two-way tables, graphs, flowcharts and formulas" (NGA & CCSSO, 2010, p. 7).

The calculation of probabilities of a "complicated situation" through the analysis of two-way tables requires quantitative data, which has to be collected in advance or simultaneous with the process mathematizing. The probability models become the generative of these data points or "data derivatives" in Amoore's term (2013), folding the future possibilities through the algorithmic calculation of risk. OECD (2013) reports, in a similar vein, "the quantification [is] for the measurement and assessment of uncertainty" (p. 35). The management of uncertain future is productive in a double sense; it generates the capacity (for people) to act in the present for the unknown future and makes new subjectivities for the kinds of people. *The body is epitomized as ability-machine conducting statistical/mathematical calculations and generating the data derivatives to enable actions in real life*. It is the positive regulatory function of school mathematics, particularly mathematical modeling, that takes the life of children as a political object to be governed what Foucault (1997) calls "biopower" (p. 247).

> Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, at, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of

center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken (NGA & CCSSO, 2010, p. 79).

In the long text above, there is no reference to the subject, but we can trace how the objective knowledge is produced by a passive ability-machine along the lines to take the control over life itself. While the uncertainty of future is taken into account, the movement between image and thought finds a material form through the relocation of algorithmic calculations of visual plots. Mathematical models do not determine the life or the subject. However, models involve a deduction and a host of techniques that make them possible. These techniques and calculations such as two-way tables, correlation coefficients or visual plots are taming the chance and domesticating not only the body in the process but also the population.

3.4.2. Models are Explanatory

Techniques and technologies for the calculation of the risk, future and uncertain point us to the explanatory characteristics of models. This characteristic of mathematical models does not only denote a simple summary of the world in a compact form. This would be merely "describing situations mathematically" (Lesh & Doerr, 2003, p. 15). In addition to description, mathematical modeling "seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based" such as "exponential growth of bacterial colonies [following] a constant reproduction rate" (NGA & CCSSO, 2010, p. 73). These explanations are not fixed or stable but reveal more dynamic illustrations for the "real world". Models are "mutable", says Heyck (2015), "but in bounded, rule-governed ways" (p. 169). As he argues, the rules and conventions (i.e. rules of calculus, projective geometry or numerical simulations) that are governing the models are necessary as they are the simplified form of reality. This is particularly important in the context of school mathematics since explanation and understanding of the world characterize what mathematical modeling is. Not to say mathematical models provide a perfect representation, but there should be a "potential correspondence" (p. 170) to function as a model in order to "develop theories and *explanations* that provide insight and understanding of the original real-world situation" (Cirillo, et. al., 2016, p. 8, my italics).

Models try to capture some form of regularity, pattern or constancy to explain the real world situation. This explanatory attribute of mathematical models, however, brings some puzzles. Although the real situations are not inherently mathematical, as Cirillo and her colleagues mentioned, they could be *explained* in mathematical terms. The search for some form of structures in the mathematical systems (Lesh & Doerr, 2003, p. 10) and in the real life makes the mathematical modeling processes bounded with the principles and rules of reason. Then, we are left with the question of what makes "mathematical modeling of the world" different than "discovering the mathematical world".

Mathematical modeling of the world reinscribes some sort of gap between real and mathematical. Models are "simplified representation" of something, which involve "scaling up or down, reducing the number of variables or components, idealizing and abstracting from the messiness of concrete situations in more familiar and tractable form" (Heyck, 2015, p. 165). The potential correspondence is necessary for the models; otherwise they cannot explain. Mathematical modeling, nevertheless, is preoccupied with the concern of representation of world, nature, social or life. This is not a representation that perfectly reflects another. However, the explanatory and relatedly predictive aspects of mathematical models create a necessary distance between the things and the words. "The idea that beings exist as individuals with inherent attributes", writes Barad (2007), "is a metaphysical presupposition that underlies the belief in [some] form of representationalism" (p. 46). In other words, if we agree with the fact that we can explain the "original real world situation" to gain "insight" and "understanding" of it, we have to assume there is some kind of regularity or pattern essentially exists in the world. This representational way of thinking or the "taken-for-granted ontological gap" between different entities in Barad's words, in effect, presumes two things: the accuracy of these mathematical models and a subject before the processes of mathematical modeling. Do mathematical models accurately explain the real world? Who is the subject in these processes? What can accurately represent its referent?

At this point, we need to turn the departure outlets of mathematical models. The "shift" towards process rather than ultimate product (i.e. one correct answer) and the consideration of system instead of the isolated individual are the potential promises of mathematical modeling. Nonetheless, the order of knowledge, the Cartesian epistemology and its representationalist triadic structure of words, knowers and things (Barad, 2007, p. 138), make the change impossible. There are systems and their complexity, uncertainty or instability is recognized. However, in our current mode of thought or the style of reasoning, in Hacking's (2002) terms, a necessary distance is required between knower (i.e. child, modeler) and known (i.e. real word, system) in order to empirically observe the particulars and make explanations, generalizations or predictions. The difference of mathematical modeling is the changing forms of objectivity and the reconfiguration of individuals as particular kinds of people with a reference to the system. The process of moving between the world and mathematical models embodies a positivistic attitude that produces objective knowledge and objectifies kinds of people. The emphasis on mathematical models and the processes function as mediators between knower and known. This mediation displays a deep suspicion for both the matter and the self but a trust for the mathematical model. We should remember how quantities and mathematical explanations provided a sense of trust and morality for the scientist or the experimental philosopher in the Enlightenment. At the same time, we should also notice the change in the form of objectivity. Mathematical models are not completely immutable like quantities. They do not constitute absolute facts, but they can potentially produce detached generalizations and ontology of associations (as argued in the previous section) that shape and fashion our actions. Historically, as Daston and Galison (1992) account, "objectivity is a multifarious, mutable thing, capable of new meanings and new symbols" (p. 123) but always embodied a kind of morality that would identify particular kinds of people through some sort of self-restrain. So, the process of mathematizing the reality, as known as mathematical modeling, is indeed a self-restrained and a domesticating that regulate the actions of collectives. Mathematical models are the new images of objectivity.

3.4.3. Models are Useful

Mathematical models are not only highlighted by "the accuracy of its predictions" and "power of [their] explanations" but also "the simplicity of [their] implementation" (Cirillo, et. al., 2016, p. 9), pointing us to the third feature of mathematical modeling: Usefulness. This characteristic is important to take into consideration since the utility is part of what makes mathematical modeling intelligible. More importantly, however, they are part of making kinds of people as contributing and effective members of the society.

The question of useful knowledge does relate back to early reforms of school mathematics in which the aim was to construct efficient and intelligent citizens. Nonetheless, the ways in which usefulness operate do differ in our contemporary times. The expansion of rationality from the individual to the system brings new understandings for efficiency and utility: "It is recognized that many of the most important cognitive objectives of mathematics instruction are conceptual systems (e.g., mathematical models) which are used to construct, describe, or explain situations in which mathematics is useful" (Lesh & Doerr, 2003, p. 31). The systems are future oriented as humans "use [these] tools to create new realities and experiences" (p. 15). The notion of creativity, in terms of planning, reconfigures the American narrative where there was a hope for a new moral and social order led by enlightened and virtuous men who were optimistic and forward-looking with a desire to shape the future (Wood, 1991, pp. 189-190). Then, the practical knowledge to be utilized in the future is not for the lone individuals but for the planning the collectives. There was a moral hope of modernity allowing the reason to inform moral judgment while it was the promise of science to solve the problems that reason had caused (Heyck, 2015). It was these "forward-looking" system theories that would hold people together. Indeed, mathematical modeling provides insights regarding that "essentially the same mathematical or statistical structure can sometimes model seemingly different situations" (NGA & CCSSO, 2010, p. 72). The predictive and explanatory characteristics of mathematical modeling conjoin with the generalizability of models in different situations, which, in fact, can be articulated as stabilization of factual knowledge and a form of rationality. Then, mathematical modeling is more than teaching and learning mathematics but to make particular kinds of people and society.

We can explain the increasing emphasis in modeling (not only in school mathematics but also as an object of scientific inquiry in social and natural sciences) through their intellectual and cultural agencies (Heyck, 2015, p. 162). A mathematical model does things; not in the sense of they are dominating scientific practices or life but providing a new kind of *useful knowledge*. They are "conceptual systems", in Lesh and Doerr's (2003) account, they can communicate, represent, generalize, manipulate and be manipulated. Mathematical models are in constant change. The dynamicity and manipulability of models make them effective and practical devices. If they do not work in particular, they could be manipulated. Indeed, what makes mathematical models intelligible is not an application of a deductive theory but its allowance of several connected situated generalizations that control and produce realities (Hacking, 1983). Modeling perspective in school mathematics, likewise, involves "developing useful ways to interpret the nature of givens, goals, possible solution paths, and patterns and regularities beneath the surface of things" where solutions of these things are the "modeling cycles in which descriptions, explanations, and predictions are gradually refined and elaborate" (Lesh & Doerr, 2003, p. 31).

The emphasis on usefulness, however, is to produce rational choices to maintain the social order where one connects "the real" and "the mathematical". This is a process to "acquire conceptual [and] procedural knowledge" where students can "transfer and apply knowledge to new situations" (NCTM, 2014, p. 9). We should note that this is a particular form of mathematical learning referring to two things that are not necessarily specific to mathematics as a subject matter. The first one is the "strategic competence", the ability to formulate, represent, and solve problems while the second is the "adaptive reasoning" which is the capacity to think logically and to justify one's thinking (p. 7). These abilities and capacities are not merely forms of school mathematics.

They shape and fashion particular forms of acting and participating in daily life. In short, they produce "cultural theses", in Popkewitz's (2008) terms, pointing to who the child is and should be. Then, emphasis on the utility is not only for the production of rational choice but also making particular kinds of people. "The finite but inventive problem solver of bounded rationality", writes Heyck (2015), "a model of man as *homo adaptivus*" (p. 82, italics original). This model of man is not perfectly rational nor he is incapable of abstraction. The ability to model the infinitely complex world enables homo adaptivus to simplify, analyze, compare, predict and design the world whereby producing rational decisions. However, these "abilities" of homo adaptivus are judged by their usefulness and viability in the real world. This is what makes this model of man "complex-but-limited adaptation machine, a bounded chooser, a finite problem solver" (Heyck, 2015, pp. 83-84). In a similar sense, as the mathematical modeling links "classroom mathematics" to "everyday life, work and decision-making" in the "process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions" (NGA & CCSSO, 2010, p. 72). This requires an adaptive but a self-restrained kind of person who can "flexibly us[e] different properties of operations and objects" (p. 6) and a body residing organically in the system in order to make this process rational. What remains, as part of the creation, is to invent increasingly precise and detailed tools for rationality and for accuracy. The question remains: How can we understand this particular form of creativity where mathematical models need to be viable in the real life?

3.4.4. Mathematical Modeling is Creative

Creativity is one of the common elements of mathematical modeling processes (Cirillo, et. al., 2016). In the course of mathematizing reality or mathematical modeling, "the students create the design specification for conducting a study [a real life problem to be investigated] that would generate the data they deemed necessary to make an informed decision" (McClain, 2003, p. 182). In this process, it is possible to create conditions for investigations without dictating the directions that students ought to take. Students are not merely consumers of data; they should be able to participate in the system with their highest autonomy. In brief, mathematical modeling of the world is expected to be a "creative" process.

As we can historicize mathematical modeling with the emergence of system theories and the shifts toward rational choice, the notion of creativity could be traced back to early Cold War years. In order to bring coherence to America's increasingly complex and diverse culture, creativity was considered as a productive and positive force (Cohen-Cole, 2014). Creativity was not a mental process nor had a genetic basis, but it was useful and productive personal trait exploded as a research domain during 1950s. The distinction between capitalism and communism was framed as a conflict between two systems that allowed freedom of thought or not. In effect, this distinction characterized American democracy as constituted by reason, tolerance and diversity and creativity (p. 39). It was a new form of civilization that could open up a room for living together. Nevertheless, the fear was to be authoritarian, conformist and close-minded person. Promoting the creative potential of the common men was for common good, which signaled the desire for prosperity as in the American narrative. Then, in this mode of comparative thought, creativity worked as a measure of individual merit as way to make social distinctions in the first place. However, the notion of creativity was not limited with the social distinctions. "The lines of influence ran in more than one direction", writes Cohen-Cole (2014), "ideals of autonomy functioned to police the boundaries of acceptable politics and social thought" (p. 62) and as a mode of daily life. Although I will revisit the notion of open-mindedness in chapter five, it is important to note that creative autonomy was not creative at all; indeed, it set the rational and reasonable boundaries for what the (im)possible is. Creativity was to inhibit the "deviant" modes of thinking in the Cold War America that could pose a threat to social stability.

The notion of creative human has its paradoxes. While real world situations are not organized and labeled for analysis and seen as a "creative process" in the interaction with natural and social world, these processes are reductive in the sense of "formulating tractable models, representing such models, and analyzing them", which depend on "mathematical expertise" (NGA & CCSSO, 2010, p. 72). It is true that students create mathematical models, but we need to think about the double function of modeling. First, they are simplified representations of the reality or explanatory devices. Creative mathematical modeling process does not have the capacity to create new mathematics but can produce the conceptual systems consisting of artifacts or tools to explain and describe the ever-changing universe (Lesh & Caylor, 2007) in its highest precision and accuracy (Lesh & Doerr, 2003). Put simply, the creation is only about inventing new tools and technologies to represent the world in highly precise and accurate ways. This is a particular and representative mode of interaction with the world, as explained before, and shares the some ideas with the Newtonian worldview emerged in the late Enlightenment as people started to think themselves superior to nature and to the rest of the world, not because they are Christian but employ a mechanical worldview (Heyck, 2015). There was a moral hope of modernity allowing the "reason" to inform judgments and it was the promise of the science to solve the problems that the reason had caused and to hold people together (p. 147). Then, creativity is a form of rationality that would provide accuracy. The more precise and accurate models are providing trust, agreement and maintaining the moral order in the social relationships. They generate a self-referential relationship with the environment, but through a distance.

The distance between knower and known brings the second function of mathematical modeling: The question of the accuracy of representations. As remembered in the Big Foot problem, for instance, the highest level of modeling cycle is the "creation" of accurate explanations with more precise measurements, which in fact configures the moral character of the person in the process and a matter of trust rather than creating new mathematics, new temporalities or new modes of thought.

If we do not think creativity as producing tools and systems for the better representation of the real life but something else, how can we explain the order of different interpretations of the same real life situation? How can we understand creativity as a ranking that suggests "initial primitive interpretations [which] tend to focus one-at-a-time on surface-level characteristics of situations" while "later and more sophisticated interpretations are more likely to emphasize deeper patterns and regularities" (Lesh & Doerr, 2003, p. 26)? How can we accept one to be creative in a self-restrained process? How one form of interpretation can be *less* creative than the other? In asking, "how students go beyond the limitations of their own initial ways of thinking " (p. 26) and supporting "habits of creating a coherent representation of the problem at hand" (NGA & CCSSO, 2010, p. 6), the process of mathematical modeling reveals no relationship with creativity but the "acquired expertise" (p. 72), which refers to the particular cultural theses for the individual (i.e. morality, trust) beneath the lines. Then, creativity is the new form of "civilization" that I have talked about earlier in the chapter while referring the reforms around 1940s and is the reinscription of Enlightenment reason and rationality. The hope to produce accurate representation of the world and scientifically based useful knowledge are in the heart of the school mathematics. But these self-restrained processes have dangers and limitations. They do not denote an "unrivaled success". They are playing an active role in making social distinctions and producing differences that simultaneously include and exclude.

3.5. "Common" Language of Mathematical Modeling for "Common" People

In this part, I continue with the shifting practices in the discursive assemblage of school mathematics along with the recognition of what has remained parallel across the reforms of these two moments of mathematics education. So far, I have unpacked the homologous elements of mathematical modeling to see how it becomes possible and intelligible in the contemporary mathematics education practices along with the its limits and dangers. What makes school mathematics, nonetheless, is more than this. There is also another component of the mathematical modeling assemblage that is about the pedagogical processes that employ a "language" that "sounds sensible and useful to ordinary people" with the aim of developing a "blue collar theory" (Lesh & Doerr, 2003, p. 8). This suggests that the task of the reforming the school mathematics is not to deliver what mathematicians are exactly doing. Instead, mathematics needs to be metamorphosed into a particular entity so that it becomes sensible, meaningful and accessible to all. This process, what Popkewitz (2004) calls as "alchemy", is a transmutation of the academic subject as it moves into the spaces of schooling. The translation processes ensemble with the governing principles of schooling and play an active role in visualizing and seeing who the child is and should be, besides teaching and learning the subject matter. They involve an embodiment of particular modes of thinking not only about the child but also the mathematics.

In mathematical modeling, we have seen the shifting emphasis on the process. This does not mean we do not have products. These include descriptions such as tables or graphs, explanations for why something seems to be true or not, justifications to pursue a procedure over another, or constructions such as mathematical models themselves (Lesh & Doerr, 2003, p. 16). They often go beyond simple numerical answers and abstract quantities but constitute standards of participation in mathematics activities. The classroom interactions become the research site. As mentioned earlier, nonetheless, they are the new images of objectivity that produce objective knowledge while objectifies the people who can produce that knowledge and produces new subjectivities. In this part, I would like to engage with these questions: How are these spaces being shaped and fashioned in a way that seem like mathematics teaching and learning mechanisms but as sites of production of particular kinds of people while abjection their others? What makes them intelligible? How can we think about these "mathematical" participations?

In the analysis of students' mathematical reasoning as acts of participations established in the classroom communities, Cobb (1999) points out the norms that are specific to mathematics and he terms as "sociomathematical norms". These norms are the forms of practices that are not merely about the psychological existence of the individual subject but more concerned with the "collective meanings and practices" (p. 28). The classroom is a learning community where the norms are co-invented to regulate the practices and to secure the exercise of power in Foucault's sense. "It [norm] is characterized less by the use of force or violence", writes Ewald (1990), "than by implicit logic that allows power to reflect upon its own strategies and clearly define its objects" (p. 139). In what follows, I elaborate on these ideas exercised in the mathematics classroom in three points of intelligibilities: First, the "common" vocabulary of the norms, which are decided by the group members, constitute a self-referential and homogenous system that individualize rather than socialize. Second, while the norms are relative entities, the constant necessity of refinement makes the "mathematical agenda" or "mathematical endpoint" an illusion. What we have is the continuous negotiations of the forms of participation and action. Lastly, norms assemble with the hopes and fears of school mathematics. Normal and abnormal kinds are generated. Dividing practices occur on the basis of inclusion.

Mathematical modeling processes employ a precise language that makes the particular engagement in the world possible. In these processes, children grasp their "obligation to explain" their ideas so that other members in the group understand the "justification for their way of structuring data" (McClain, 2003, p. 184). Explanations and justifications are necessary parts of mathematical modeling not only because of they constitute mathematics teaching and learning but also they facilitate the process of getting connected with one another. They are the part of the communication mechanisms in the classroom in addition to the learning the subject matter. Through the way children organize the quantified entities to observe and engage with the world, they generate "taken-as-shared basis for communication" (Yackel & Cobb, 1996, p. 467). These practices reveal a communicative system where both teacher and students actively participate and constitute what counts as acceptable explanation for the phenomena they are mathematizing. These acceptable explanations and justifications are co-constructed with teachers and students and named as sociomathematical norms that provide the group to meet in "a common denominator" with a reference to self (Ewald, 1990). This communicative system is self-referential. Everyone can find a way to measure, evaluate and identify themselves in relation to the sociomathematical norms that are established together.

Nobody conforms themselves to the external rules, they rather compare themselves to one another with respect to those "taken-as-shared" practices. Indeed, "effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and *comparing* student approaches and arguments" (NCTM, 2014, p. 10, my italics). The key aspect for the teacher is to capitalize these approaches provided by the students to "optimize" the chances of learning, which helps to initiate shifts in students' diverse ways of reasoning toward more efficient and sophisticated yet common solutions for the problems. This is exactly the positive control of normalization circulating around a principle of comparison. As Ewald (1990) contends, "the norm is equalizing, it makes each individual comparable to all others [and] it provides a standard of measurement" (p. 154). Then, invention of a common language is to accommodate and adapt diverse ways of thinking into a homogenous system where the norms invites us to imagine ourselves in comparison to others with a reference to the system that we have shaped together.

If we follow Ewald's arguments on the norms, we should also note that that norms are not totalitarian, they allow a space for us to live our individual lives yet nobody could escape from those commonalities. A similar process exists in the practices of school mathematics where the establishment of sociomathematical norms is to foster "intellectual autonomy" (Yackel & Cobb, 1996, p. 473). Nevertheless, we are left with a paradox: While these norms foster intellectual autonomy, "what becomes mathematically normative in a classroom is constrained by the goals, beliefs, suppositions, and assumptions of the classroom participant" (p. 460). Then, in order to create a homogenous space of collective meanings, the beliefs and values of individuals need to be cultivated in particular ways to maximize their "learning" opportunities. All these components, sociomathematical norms, goals and beliefs, interact with one another and develop in harmonious ways so that they constitute together "a dynamic system" that individualize in relation to co-established common standards rather than socialize. That is, as I have argued earlier, an adaptive but self-restrained kind of person is fabricated through these formations. Self-referential and communicative characteristics of norms of the mathematical activities provide another layer for these fabrication processes in addition to the historically constituted notions of rationality and accuracy for the modeling of the world.

It is important to note that the norms are not fixed, stable or an external set of practices. Norms are specified in terms of time and space; that is, they cannot exist for indefinite periods (Ewald, 1990). However, this does not mean that norms do not have adhesive characteristics of societies that constitute the complex network of social, cultural and political relations. They provide a self-referential standard of measurement for a group to meet in a common ground. The local nature of the norms open up space for negotiation to find a stable yet agreed upon basis. In a similar vein, there is the negotiation of norms for productive mathematical environments to ensure the "mathematical validity" of the discussions in modeling activities (McClain, 2003, p. 177) rather than imposed set of practices in order to optimize the chances of growing would occur without dictating the directions (Lesh & Doerr, 2003). Nonetheless, in order to generate this "taken-as-shared" knowledge, students engage in lengthy discussions that are concentrated on the "ways of structuring the data and models or inscriptions that could 'best' be used to validate the argument" (McClain, 2003, p. 184). Then, the negotiation processes become a course about agreement and establishment of a stable group. As Ewald (1990) argues, "the achievement of specific ends is less important than maintenance and negotiation of [stable social] state [in a normative society]" (p. 158). Then working on the organization of a common language and communication norms loses those "mathematical endpoints" that is aimed initially. A mathematical agenda of these discussions become an illusion in which individuals are situated in "the process of deciding between truth and untruth of the mathematical narrative" (Wood, 2016, p. 331). That is, what remains at the stake of these practices is to test the validity of the argument through application of quantities, rather than inventing or constructing the mathematics. Then, "obligation to justify" (i.e. Cobb, 1999, p. 16) the arguments is not a mathematical objective but a "technology of trust" in Porter's words and a tactic for governing the child as a moral member of not only the classroom but also the society.

The normative account of mathematical identity that requires particular ways of participation in the mathematical communities makes it possible the existence of the classroom members. Some of the mathematical models created by the students are identified "less clear and cumbersome" (McClain, 2003, p. 185) or "less persuasive" (p. 183) not because of the mathematics in itself but they less effective in making decisions. This is seen as detrimental to growth since those students are no longer members of the classroom community. As Cobb (1999) claims, "one of our primary concerns [...] is to ensure that all students are 'in the game'" (p. 35). This has to do with the collective belonging to the classroom community and also effective participation to "maintain a just, democratic society" in the broader respects (p. 36). While these normative accounts regulate and produce particular subjectivities (Butler, 1990), such as effective and productive member of the society, they simultaneously generate spaces for "others" who seem "cumbersome" in those negotiation processes. This has less to do with the mathematics as a subject matter but the *ability* to make persuasive arguments and justifications. The division between bodies as normal and abnormal occurs on the basis of community building.

The principles of participation in mathematical modeling processes to generate these products organize the classroom as a collective of mathematically able bodies and a form of cultural complex that coordinates the human activities. These collectives are not a matter of individual psychologies or mental states, but they are communities of mathematical doers that can be considered as analogous to Daston's (1995) moral economy. Organization of the mathematics activities shapes and fashions how individuals act and participate, and ultimately constitutes not only the products of mathematical modeling but also the kinds of people. As Cobb and his friends (2009) mention, it is this "moral dimension" of the mathematics classroom that make "identities" tractable for analysis (p. 47). Then, these processes are not merely about the teaching and learning the content but also reveal the classroom site as a space for making the child as moral agent and their particular engagement with the world. We should remember what Daston (1995) says about the principles of the quantification practices in the moral economies are intolerant of "deviants" who do not objectively find the close fit between mathematics and the phenomena that is explained. Then, the viability of mathematical identities, either normative or personal, can be only understood as children participate in the moral dimension of the classroom, which requires the re-imagination of mathematics as it enters to the school spaces. These participations are translated into socio-psychological qualities such as "decision maker", "effective student" or "problem solver". Understanding these participations and subjectivities occurred during mathematical modeling processes requires "go[ing] beyond investigating typical development in natural environments" toward "focus[ing] on induced development within carefully controlled and mathematically enriched environments" in order to optimize the chances of these developments (Lesh & Doerr, 2003, p. 22). For effective participation in the mathematical modeling processes, a learning space needs to be created in a way that facilitates the particular involvement and engagement in the world through the mathematized entities. However, the involvement of mathematically able bodies in these spaces occurs on the basis of the abjection of the others.

3.6. Final Remarks on Contemporary Reforms and Practices

In this part, my focus was on the contemporary practices of the discursive assemblage of school mathematics. Mathematical modeling, at the first glance, seems to engage new ways of thinking about the world, mathematics and the self. There are also changes in the discursive practices that make this practice possible. As I have described, some of these discursive shifts are focusing on rational decision, anticipatory logics and the modifications in the calculation of risk. The 21st century reforms on school mathematics have taken our attention to the new practices that have pointed out the classroom community as a site of investigation with an emphasis on the processes rather than products. These have raised some novel notions such as collaboration, participation and individual autonomy and configured new forms of subjectivities such as flexible people, adaptive kinds or decision makers.

These changes and shifts, nevertheless, do not provide new ways of thinking. Reform initiatives, in fact, re-bring the Enlightenment reason and rationality and re-inscribe the mechanisms of exclusion. In this configuration of contemporary practices, while different forms of power start to operate in new domains, there are still axes of differentiation and normalizations, which should be understood in correlative mechanisms rather than cause effect relations. The more the child participates in the classroom community or the more accurate models he makes, the more he becomes "mathematically" successful, competent student. The normalization occurs, according to Foucault (2007), in establishing interplay between these different distributions of normal (i.e. members of the classroom community, different models), not referencing oneself to the one specific external norm.

4. Concluding Thoughts on "Mathematics"

Is there mathematics in the nature experienced by humans or is mathematics determined by the human mind? Neither. Mathematics emerged as a cultural-historical practice that makes particular reasoning about the nature and space possible. I have argued that this is a particular mode of reasoning about the world, and it is bounded with the representations and their accuracies. The continuities such as "mathematical precision", "accuracy" or "numerical validity" have operated as a communication technology between the world and the self and a tactic for governing the child as a moral and productive member of not only the classroom but also the society.

While this representational thinking of the world is a form of cultivating the moral conduct individuals, it is, at the same time, operating as a comparison mechanism to distinguish two human kinds: Those who tell the truth as moral subjects through using the language of numbers and mathematical tools and their Others. Then, although in different arrangements and different enunciations, the representationalist triadic structure of words, knowers and things (Barad, 2007, p. 138), make the change impossible. It becomes only the distinguishing of the particular kinds of people who are able to do that kind of quantitative inquiries and to make rational explanations through this language with the purpose of being "true" to nature. Put differently, when the discursive lines, segments and practices are examined more closely as an assemblage, it is not the mathematics that is the site of intervention but more about the desire to make mathematically able bodies as moral subjects.

The contemporary practices operate as particular solutions and plans for actions to secure the uncertain futures, these enactments assemble with historical ideas, institutions and technologies, go beyond the initial intentions and aims and become a mechanism that secures power relations. At this point, let me be clear. The historically assembled practices do not mean the exact re-writing of the past. That is, there is a need to think of the discursive assemblage of school mathematics moving along different layers, relating back and forth; but extending itself in a way to keep transforming but with new technologies and tactics of normalizations. The task is to examine how these discursive practices are organized and are enacting differently in a specific time and space but along a similar horizon. Then, these enactments could be explained in a spiral set of connections in which mathematically able body is a spatiotemporal configuration for the self and for the society. The particular enactment of the discursive assemblage of school mathematics produces a domain of cultural intelligibility where the mathematically able bodies are viable and their Others become the abject. In this chapter, I have encountered with the "mathematics" of this domain. Next, my focus will be on the ability.

Chapter IV

Denaturalizing The Developmental Narratives on Ability: From Mathematical Maturity to Mathematical Learning Trajectories

1. Introduction

The historical analysis of the two moments in the mathematics education practices makes the developmental reasoning visible while it reveals a transmutation from “mathematical maturity” to “mathematical learning trajectories” in the enactment of what counted as mathematical ability. The whole corpus of “scientific” knowledge that organize the mathematics education practices, including learning, teaching and research, is produced by taking “developmental reasoning” into account as something given or something commonsensically natural in not only organizing the mathematics education but also making up people. Rather than approaching this issue as something that has to be change, my purpose in this chapter is to historically problematize these narratives on the human race. That is, the aim is to denaturalize the developmental discourse on mind and relatedly human by unraveling the historical and social practices that make it possible and seemingly reasonable.

In the context of which developmental narratives regarding the mind and relatedly humans seem unquestionably natural, a re-examination of the sciences of mind becomes important due to two main reasons. First, mind is commonsensically assumed as an object of research in the course of 20th and 21st century scientific practices; second, mind becomes elevated as “an administrative platform” and “plane of composition” in institutional life. That is, the hopes and fears for a different future in connection with the nation building projects territorialized in the mind (Baker, 2013, pp. 3-4).

In what follows, I organize these two moments of mathematics education according to their own identification processes, articulations and solution methods for those who are "not-yet-developed". These technologies are embedded in a redemptive discourse of science, they become instrumental to distinguish between two different kinds, those who are developed and those who are yet to developed, in the developmental continuum. Although new techno-scientific ideas (i.e. constructivism) are brought into being as solutions for the problems produced by the previous practices, they become another enunciation of the historical questions while extending the scope.

2. Mathematical Maturity

In the previous chapter, I have started to talk about the happenings during pre-post WWII years with the mathematics education through the discursive assemblage of school mathematics. As a commonsensical practice, mathematics is referred to as a form of civility, progress and development. I have examined the so-called mathematical virtues (i.e. precision, accuracy) in terms of what they do, what their limits and dangers are. Another dimension together with these practices was the continuous theme of "mathematical maturity" that implied a developmental reasoning circulates throughout this discursive assemblage, which incorporated political anxieties about culture, nation and well-being of the population. The authorization of the fear of "savages", "primitive societies" or "anti-democratic regimes" enabled a continuum of a developmental reasoning that differentiates people and put them in a hierarchy of standards. In this part, I expand what makes these discourses possible and how they become reasonable.

2.1. Redemptive Enactments of Developmental Narratives

The enactment of "discovering the mathematical world" entails a developmental reasoning about knowledge, people and societies: The accumulation of the mathematical discoveries suggests parallels with the developed nations, cultures and men. According to the Committee on the Function of Mathematics in General Education (1940), for example, men's mind tends to follow similar patters with the "discoverer" of mathematics or science like Newton or Leibniz. Students should gradually develop an always "more mature understandings" and they acquire "richer and more discriminating content" in their study of mathematical progress (p. 59). The cumulative characterization of mathematical knowledge was considered to be "applicable" to the students' learning mathematics in schools. The role of mathematics education was to cultivate the ability to generalize from "premature hunches" to "mature logical thinking" (p. 194). The "highest stage of abstraction" had to be compatible with the "perfectly rational universe" for progress and growth of the culture, nation and population.

Developmental narratives, as Baker (1999) puts, view human life where the abilities unfold in a set of steps to be acquired in a series of stages. In the pre-post war period, the characterization of upper stage of ability, reasoning, thinking for mathematics was arising from how the universe was seen inherently rational. That is, mathematics education was to be organized around stages that were leading towards abstract knowledge (Lynch, 1939). The mathematics program for the post-war country needed to be planned to enable pupils to achieve "mathematical maturity" and "power" (NCTM, 1945). While these practices and plans were important in terms of the nation building projects they authorized a developmental reasoning for the human race.

2.1.1. "Man of Culture" in the Developmental Continuum

A developmental logic operates along a discursive line that makes the idea of "mathematical maturity" possible through embracing a continuum of values to know the human race and the differences from one another. Two interrelated dimensions made these hierarchical scales "scientifically" possible and were part of the body of knowledge about the "developing" child: The emergence of modern linear time and the evolutionary logic for living beings.

The Enlightenment thought, according to Johannes Fabian (2014), marks a break with Judeo-Christian vision of Time in terms of a history of salvation to the one that resulted in secularization of Time as natural history. Although this does not entail a significant change from sacred to secularized, it is productive in understanding the "nature" of human life as a linear experience in the form of civilization, evolution, development, acculturation, and modernization. The temporal yet accumulated view of human nature was preoccupied with the transcendental achievement of reason and rationality such as "mathematical" or "rational" universe. The enunciation of reason in the discursive assemblage of school mathematics during the pre-post war years was, similarly, to stabilize the future by eliminating chance and to naturalize the "growth" of mathematical maturity. This stabilization of future and naturalization of growth reveals a particular view for the development of human as a process between savage and civil. "Mathematics is mirror of civilization" was not an empty slogan or merely an idea, but enabled a further characterization of the making up people in a hierarchical scale from savage to civilized, from primitive to developed. Introduction of the modern Time into the living world translated all living beings from simple towards complex and linked to a single history describing a common generation (Rose, 1985).

"Man of culture", as reported in the post-war plans of NCTM (1944), was consisting of those who have a control of algebra as a language, a concept of proof for sound thinking and a tool for problem solving. While the hope was to cultivate the abilities for mature logical thinking of the child, it simultaneously authorized the fear of being primitive without those qualities. In addition to these hopes and fears, these statements reveal images for the Man as a machinery to perceive stability and to analyze the child in motion and as an organism responding to environment (Baker, 1999). That is, "without a view of Man" as a project to be unfolded as series of sequences, "the developing child" would not be possible as a research and reform site (p. 811).

The formation of the body of knowledge, which embodied developmental logic, was not an automatic application of savage-civilized binary. Parallel to the processes of secularization time, theological understanding of human nature was replaced with the science (Wynter, 1995). The human mind became something empirically observable, verifiable and testable. That is to say, mind emerges as an object of science as an analytical category. In effect, these modern inventions made possible the belief that each individual human's growth recapitulates the stages of evolution of race. In mathematics education, the cultural modes were distinguished in terms of stages of mathematical maturity such as immature, premature or mature based on the degree of correspondence with the taken granted notion of "mathematical world". The linear and chronological movement from premature to mature (or from the savage to the civilized) makes the evolutionary temporalizing possible (Fabian, 2014), particularly in mathematics education practices with the notion of mathematical maturity. As Baker (1999) argues a child's development was described as moving from savagery to civilization transmogrified through "culture epochs" of "racial evolution" that corresponded to "stages of development."

Nevertheless, what enabled the developmental reasoning about the mathematical maturity was not the science itself, but the salvation narratives embedded in the enactments of science. It was the task of the science and the reason to "rescue" human subjects from their immature spaces. Although symbolic representational system of Judeo-Christianity was replaced by its secular variants during the "epochal shift" in the ways of thinking, the redemptive themes have continued to exist in the processes of "development" in Western societies (Wynter, 1995). That is, a simultaneous process for the enslavement and rational redemption of human nature was to be achieved through reason and rationality, instead of spiritual and eternal salvation of the feudal-church order. Obscured in this emerging commonsense, the mind was named a site of research to know the human nature and to configure the redemptive narratives for the human soul. As Popkewitz (2008) argues, the forms of reason and rationality visualized the civilized through these concurrent uniting and dividing practices that organize the notions of "race". The construction of the race of the nation as a "unified whole" enables a developmental continuum where the one end is the presumed civilized human type with a shared biological heritage and the other end is classified as "uncivilized" and dangerous for that unity (p. 38). The project of making the "American race" was also visible in the reform efforts for the school mathematics.

> We should make sure that our youth shall have the chance to participate intelligently in protecting and bettering the heritage we shall leave them. Furthermore, our youth should be given the vision of what the America of tomorrow may and should become (Bond, 1942, p. 372).

Preserving the American race and making it better was possible through the developmental logic entailed in these statements. That is, the assumed "civilized" heritage was to be protected from those who might be dangerous to that; also, savages were to be rescued and to be developed to make them "civilized" so that *all* would contribute to the unity of the American race. This was what schools should do for further defense during the WWII. Of course, these practices were productive not only in a sense of a patriotic duty, but also they enable to distinguish two races in this social-discursive complex. The assumed space for the collective belonging of all included those who are developed and fit to that "civilized heritage" and those who are not-yet-developed would become the biggest problems within the time.

2.1.2. Nation Formation: Leaders of Democracy and Peace

The desired highest stage of mathematical abstraction was also occupied with the nation building projects given the emergencies and uncertainties of the war. This dimension was vastly related to the cultivation and preserving of the civilized heritage that was compatible with the "rational" universe, which produce racializing and sexualizing discourses. Nevertheless, it generated further thoughts about what it means to belong a nation, to protect it and to maintain the "democratic" order.

The mode of thinking was the ability to do precise and careful work in the program of winning the war. This continued after the war, too. As mentioned in the previous chapter, precision of thinking and accuracy of the work was related to the processes of making the moral subject who was true to nature. Considering these "abilities" in the context of developmental reasoning, nevertheless, opens up a new segment in the discursive assemblage of school mathematics.

Construction and protection of the American race became an important issue for "progress" since the world was "degenerating" with the anti-democratic regimes during the WWII. This fear of degeneration was important in two points. First, the "United America" had to battle for its very existence and gird the people thoroughly with precise and careful thinking. These were internal concerns, within the nation, to safeguard the heritage as a nation-building project and strengthening the collective bonds.

> We need to dedicate all our time, powers and property to the task of maintaining and improving the splendid culture we have built so that we will be in a position to enforce the inclusions of the ideals of individual security and freedom when world order replaces present chaos (Bond, 1942, p. 372).

Second, given the environment of chaos, war and unpredictability across the globe, it was important for mathematics educators to make pupils as "loyal Americans" who could use mathematics to solve the social problems all over the world. The enrichment of living through solving the social problems in the contingencies of the war was not only built into the love for the country but also the working for "human betterment" across the earth. The United States had to be allies with the countries with urgent "need" of food, energy, technology and democracy. However, this had to do with establishing and maintaining the moral order in particular ways across the globe.

> [The need of our country] is for more resoluteness in facing the future- more assurance that we shall do our part to make this a better world in which to live and work. We need a better morale. The teachers of mathematics should join with others to overcome fear and lack of confidence in our ability to finish the job with charity for other peoples and without hatred for any (Bond, 1942, p. 376).

Thinking these points together, the humanitarian gestures put the nation in a position of an "external struggle" where the outcomes were depended on their "internal fitness" (Rose, 1985). If the America would become the "world power", this needed to be mediated through the internal social arrangements in relation to the competitors.

> What America will be in the 50's and the 60's of this century will depend largely upon *the intelligence, the moral stamina* and *the heroic stalwartness* that our youth of today take to those years. Our national life will depend in no small measure upon what happens in our classrooms during the 40's. Hence our schools should be sustained in their effort to increase the precision of thinking and the accuracy of work of our youth (Bond, 1942, p. 372, my italics).

The redemptive discourse embedded in these statements reinstalls the colonial way of thinking in a continuum from savage to civil. This required inserting new arrangements to make hierarchical categories of mind reasonable and legitimate; for example, working for "human betterment" as "loyal Americans" was one of them. However, the enactments of the developmental reasoning authorized the distinctions between children in a hierarchical spectrum from slow learner to bright pupil. These practices generate a moral space with two facets: Those who had to "be saved" and those who were to be the "saviors". The selection of the "bright pupils" as future leaders of democracy was the other duty that mathematics educators had to encounter (Moore, 1941). It was vital for the school system to "provide the maximum development of the powers and abilities of the bright pupil than of the mentally slower one" because "democracy [was] being assailed from within and without the borders [of the United States] by advocates of fascism, Nazism or Communism" (p. 155). A distinction between bright and slow pupil was made to save the "democracy". Adequate provision for the bright pupil was necessary to plan the future of not only the United States but also across the world. It was those "bright pupils" that would give the "dignity" and "nobility" to the human life. Their learning opportunities needed to be maximized:

> In order that their full capacities may be developed, superior pupils should have superior teachers. The teacher should be a distinctly intelligent person… Though such a teacher should manifest an awareness of current problems and take an interest in them, he should also reveal a firm attachment to those great underlying achievement and interests that give dignity and nobility human life (PEA, 1940, p. 137).

"The struggle of American exceptionalism", writes Popkewitz (2008, p. 48), incorporates both Enlightenment and religious images where "the civilizing mission" of the nation is to spread and to disseminate the ideals of a unified enlightened humanity. While the idea of civilization of the human kind was important to make rational citizen, that includes the *all*, who had to maintain the social and moral order in the daily life, it was possible through these redemptive narratives, which made a doublet of human kinds. The identification of the "bright pupil" reinstalled the civilizing machinery between darkness and light. It was the task of "bright pupils", the future leaders of "democracy", to "give" the dignity to human life that was "suffering" from "backward" conditions.

2.1.3. "The most able pupils"

School mathematics emerged through the relations produced in the conjunction of these "developmental" practices that formed a hierarchical continuum from the darkness to the light, from the savage to the civilized or from the premature stage to the maturity in mathematics. Inserting the "democracy" in the discursive assemblage of mathematics was important to arrange the governmental rationalities to make the self-directed citizens to maintain the social order in the daily life (which will be the focus of next chapter). However, the desire to maintain "the democratic order" revealed another dimension that made possible the developmental narratives circulating in this discursive assemblage. That is, in order to secure the democracy, "the most able pupils" or the "pupils with able minds" (PEA, 1940, p. 146) had to come to the fore to "be train[ed] as wise, competent and just leaders, capable of reasoning and solving difficult problems confronting [the] nation" (Moore, 1941, p. 157). In effect, the differentiation of "the most able pupils" was going to make the homogenous ability grouping possible and reasonable (PEA, 1940, 126).

Naturalizing the developmental stages through evolution of human race was conjoined with the "patriotic duty" to make the able pupils as safeguards and leaders of their country. However, "the cultivation of the ability" was not directly about learning mathematics, it was rather "ability to generalize and the study of the values and dangers of induction" (PEA, 1940, p. 194). Making inferences and inductive reasoning as the highest stage of mind connected with the cultivation of human reason and rationality that was stabilized with the desire to objective representation the world.

> Arriving at conclusions involves the characteristic process of rational thought, namely inference. Inference is a typical intellectual process, ranging *from an almost immediate and impulsive realization*, as when a child infers its mother intentions, *to highly analytic and subtle behavior*, as when a detective solves a murder mystery or a philosopher develops a metaphysical formulation of the world order. Formal mathematics is largely concerned with drawing conclusions from premises within a specialized field (PEA, 1940, p. 187).

The "reason" of these statements suggests two points. First, the ability to do "formal mathematics" is not specific to mathematics but it is more a reinscription of particular way of doing science such as making inferences and drawing conclusions about the world. Second, a developmental epistemology makes the statements about school mathematics possible. A continuum is ranged from "immediate and impulsive realization" to "the highly analytic and subtle behavior". The achievement of mathematical maturity was possible through the development of reason and rationality in these processes of application or inference, which is necessary for "effective transfer" (NCTM, 1945, p. 205). "The most able pupils" were capable of using mathematics, which did not necessarily suggest an ability that was specific to mathematics but an ability to transfer the knowledge, which constructed "a hierarchy of excellence" (Danziger, 1997, p. 9).

The fears of degeneration or backwardness created a psychological domain for those who were not designated as the 'able', and they had to be saved by the leaders of America or by the most able ones. This was not merely an external struggle between countries during the war to disseminate the "civilized heritage" and "democracy" across the world. Internal arrangements had to be done to conserve this heritage and ways of living. It simultaneously became a question of population. The task became thinking about the intelligence of the population to plan the future of America. "Slow pupils", who revealed signs to distort the internal fitness, had to be "rescued" and "developed" to make them as contributing members of society. The ontological separation between slow and bright pupils through developmental epistemology authorized a necessity and a possibility to equip children with "knowledge" to make them "able" for effective transfer.

The developmental reasoning that is tied with the regular and irreversible time made possible to think "scientifically" about the "possession" of this knowledge by human beings. Mind became a site of scientific representation and intervention to overcome the internal and external fears of the country. The enactment of scientific discourse on the (re)forming the mind, which is more than ideas but a set of instruments, tools and techniques, has made the truth claims about human being possible (Rose, 1985).

2.2. Making Visible the Different Kinds: Sciences of Mind during Pre-Post WWII Period

In an address given at the Joint Meeting of the NCTM and the Department of Secondary Education, the matter of concern was "the slow pupil" and his physical, emotional and mental characteristics (Eisner, 1939). A necessary adjustment was required in the "administration machinery" by the segregation of "the doubtful cases" (p. 15), which were making an unsatisfactory progress. While the identification of the "bright pupil" simultaneously catalyzed the processes of the differentiation from their others, the social and political anxieties regarding the "slow pupil" was assembled and connected with the multiplicity of discursive lines and practices that made the "adjusted" outline of the tracked courses was possible and reasonable. "Slowly maturing pupil" became visible quantitatively such as the time of task or by measuring their mental abilities. Nonetheless, slow pupil did not become the object of research and teaching only because of the pure development of the "mathematics ability", but rather it was a fear of degeneracy given the declining numbers regarding the intelligence of population. The desire to make the world a better place was possible by solving the "problem" of the slow pupil. That is, the scientific practices of mind were a question of making the society by making the child.

2.2.1. The Evolutionary Discourse on Mind

The attention of "bright pupil" and his "superior" distinctions were important for homogenous ability grouping; however, the founding moments of the psychology of mathematics for instruction always concerned with the pathological, the "slow pupil", who was to be known through the rational procedures and normalized with the reformatory tactics (Rose, 1985). The stages for "mathematical maturity" were produced in the sense of movement from the "premature hunches" to the "mature logical thinking", allowing a developmental epistemology and educability of those premature or immature minds. Nonetheless, this movement was not about making everyone mathematically mature, but about the multiple processes of normalization for the "slower" ones. The "slow pupil" had to be known where "the motive was to gain information which might be useful in the fields of diagnosis, remediation, guidance and possible prediction of success" (Stein, 1943, p. 164). As reported in the Mathematics in the General Education Report (1940), knowing "the deficiencies" of "dull" or "slowly maturing pupil", would "make it possible to select appropriate material and carry on instruction far more successfully" (p. 134).

Mathematical maturity, in principle, was a reiteration of an evolutionary discourse for the development of mind from savage to the civil man. The exercise of this discursive line, however, differed. While "mental powers" were initiated to differentiate between species including humans, as suggested by Darwin, the concept was extended to see the differences among human groups (Danziger, 1997). At the same time, the evolutionary logic was not limited with the individual mental capacities of people. The essential point about minds and organisms was the degree of adaptation to their environment in which the better adaptation was correlated with the higher level of intelligence. These adaptation processes, which connected with the modern linear and accumulated construction of Time, revealed a graded series of actions and behaviors (pp. 67-69). Although this developmental clock-time and biological productivity resonated the American narrative of progress (Lesko, 2012), it simultaneously exercised the belief that each individual human's growth recapitulates the stages of evolution of race. As Danziger points out, intelligence is "an inherently graded biological characteristics" (p. 70) that rationalize the distinctions between human groups, invariably distinguished by race, gender and social class, in a hierarchy of excellence.

However, while the maturity or the mental power suggested a continuum of values and a degree of adaptation to the environment that could be ranked, they were not concrete enough to be classified. Intelligence, as a degree of environmental adaptation, was an abstract idea by itself. Who was the mature enough? Who was less mature or the least mature one? Did the slowness indicate less adaptation to the environment? Where was the place of intelligence in this matrix? The offering of different tracks accelerated the urgency of the problem of differentiation between the groups and selection of different materials for distinguished curricula. As the examination of pupils' mathematical ability became necessary to track them, standardized intelligence tests emerged as one of the measurement methods for ability (MacNeish, 1941).

In the pre-post war period, it seemed a fairly "safe assumption" that "slowness," or inability to understand and apply mathematical concepts and relationships was a function of general intelligence (Eisner, 1939, p. 9). Nonetheless, intelligence should not be purely determined by the biological heritage. While part of its possibility was the degree of biological adaptation of organisms to their environment, the developmental epistemology made it possible as something "improved" over time. Indeed, a majority of published statements preferred to use "slow pupil" rather than "mathematical moron" whose low level of ability stem from the inadequate instruction (Lee, 1947, 294). If it was a merely biological category, what could be the role of education? What would be the point of knowing and separating the slow pupil?

2.2.2. Scientific Machinery of the Mind: Political Economy, State and Population

Nineteenth century industrial capitalism, as Danziger (1997) points out, could be remembered as a major breakthrough in the changing the conditions and practices of educational systems. Schools came to be seen as an effective technology that would prepare the children for new conditions of adult life and industrial work. In this course of transformation of schools into an effective technology, the school day was divided into timed lessons, examinations became formal and written, and books and syllabi were standardized. However, the processes of educational activities were not taking into account unlike our contemporary practices. That is, the educational matrix was established around an input-output mechanism. In this machinery, unpleasant outputs were directly the result of inputs such as "the poor quality of raw material" or "the innate endowment of pupils" (p. 78). The differential values of outcomes automatically reinforced the differences between pupils: Bright, superior, dull, slow and so on. Although the notions of superiority and inferiority were in assemblage with the assumed biological heritage of the human beings, at the same, the discourse had to be rational, too. In this climate, the standardized intelligence tests were "psychology's great gift" to humanity to rationalize these "differences" of pupils so that schooling became possible - a "standardized" and "universally valid" device to prepare children for an average adult life configured by the demands of industrialization (pp. 74-77).

Nevertheless, these administration processes, inspection and evaluation of conduct and capacities were to make kinds of people through the had to be rationalized through the invention of technologies and tactics in order to provide extensive and detailed information about the whole population from a single intelligence test score. As Rose (1985) puts, If we were to be able to exercise control over the apparently random, yet evolutionary crucial, processes of individual variation within species we had first be able to grasp them, to conceptualise them in order to be able to operate upon them (p. 68).

The evolutionary discourse, for Darwin, was about the relations between individuals within the population; that is, the biological survival depended on the ability to reproduce and accumulate the conditions. When translating these into the social milieu, population came to represent an organic unity for constituent individuals where each of them contributed to the average characteristics of the whole while preserving the negligible variations (Rose, 1985).

This points us to a move from strictly determined equations into the probabilistic regularities in the world. The avalanche of the printed numbers, as argued by Hacking (1990), replaced the universal law of nature applied to all humans with the statistical distribution of the values, which would make possible the category of "normal people". Nonetheless, this was not a total erasure of determinism because the aim was to look at the regularities through applying the "law of large numbers". That is, the more people get enumerated, the more stabilized the distributions are. Then, the existence of "normal people" was not by chance, rather through procedures, tactics and rationalities that "tame the chance".

These practices, at the same time, were productive in two related senses. First, the application of the law of large numbers onto the human sphere created the category of "normal people". Second, it enabled a mechanism to act on people as objects of intervention and governance. The notion of "slow pupil" emerged as a category to be acted upon not the individuals but upon the populations to make up people and the society.

2.2.3. Population as Object of Governance in the Sciences of Mind

Population became an object of political practices and governmental policies through the concern of the "well-being" of the population(s), consisting of the individuals who made it up (Rose, 1985). The fears of degeneracy of the population, in fact, were another way of posing the question of moral order through the individuals who deviated from what designated as norm. For example, it was not the eugenics movement itself resulted in the catastrophic effects, but the social and theoretical conditions (i.e. statistical distribution of the individuals and those deviated from the "normal") that made this strategy both possible and significant (Rose, 1985).

In these larger political and social transformations, the "problem" of slow pupil emerges by the changing characteristics of the population in the schools. When the average for intelligence test scores was calculated for the students coming from upper economic level of society, it did not match with the average of unselected mass of secondary pupils. "Slow pupils", "backwards" or "defective children" was made visible in the network of mathematics education practices, since the current system was being "dishonest" to the higher-achieving students (Reeve, 1940, pp. 123-125). As an effect of these different discourses a general consensus was established that *standardized intelligence tests* as reliable methods of measuring the mathematical ability. Now, unlike the initial practical purposes, individual's score on an intelligence test was a real entity that could be measured like height or weight and became a pervasive technology applicable to entire population (Danziger, 1997).

The group or the population that the slow pupils belonged had to be known in order to "develop" them and to make their scores closer to the average intelligence of the whole population of the country. The cumulative record of the pupil was gathered through the introspective devices.

These included but were not limited to the scores of intelligence tests, standardized achievement tests, school marks, teacher judgments, age, physical defects, special abilities and interests, vocational plans, economic status and occupation of parents and language spoken at home (Reeve, 1940, p. 128). Things went beyond their early intentions. The ability in reading was underlined. This emphasis was less related to time on the mathematical tasks, but to monitor the changes of school population through identifying the children who do not speak English in their homes. Another line of difference between children was recognized. Populational reasoning was emerged in the discursive assemblage of school mathematics. Children, who performed below the average, were populated into groups in terms of American-born parents or not. The point was not only making their mathematics performance at least average, but also the efforts were concerned with "intelligent adjustment [of the slow pupil] in the present day world" (p. 141). The development of "slow pupil" was to make a better citizen who could qualify the average standards; it was a nation-building project through populational reasoning. Post-War Plans of school mathematics were clear: "[the problem of slow learner] is crucial in our land. It calls for a different curriculum and a new approach. It demands a new program in mathematics that we should provide" (NCTM, 1944, p. 230).

2.3. "Slow Pupil" as an Object of Psychology of Mathematics for Instruction

"The problem of slow pupil" was one of the stimulating questions in the field of mathematics education. Knowing the "slow pupil" was not enough, his or her differences had to be accommodated, the instruction had to be modified so that they were going to be assimilated into the established culture. There should not be a risk of disorder both in the mathematics classroom and in the society. These people, at the end, were going to live together. The "slow" ones should not interrupt the fabric of the modern life. While the assimilative practices shared the evolutionary discourse of environmental adaptation, the developmental logic made the mind a thing that could grow over time to catch the standards of the average. Representing the mind of the pupil in a hierarchical ranking stimulated a desire to intervene. And this "ambitious" question came at the Joint Meeting of the NCTM and the Department of Secondary Education: "What then has mathematics to offer to these pupils that will be of genuine value to them and by what methods may the offering be best assimilated by them?" (Eisner, 1939, p. 10)

While this address, as I have mentioned earlier, was concerned largely about making visible the "slow pupil" and the tracking them into content-specific classes such as "social mathematics", which shall be discussed in the next chapter as well, these questions simultaneously revealed a desire to develop a body of "scientific" knowledge of the instructional practices, methods and solutions for the slow pupil. The drive for the pedagogical application of the psychological research had mathematics educators search about the "mixture of experience and intellect makes that thing called *mathematical ability* happen" (Resnick & Ford, 1981, p. 3, italics original). In the 1930s, mathematics educators started to question the application of "universal, scientifically derived facts" from psychological experiments directly to the instruction regardless of subject matter (p. 5). Although one had to wait until the New Math period (late 1950s-1960s) to see the proliferation of cognitive research on mathematical thinking, reasoning and problem solving, the initial attempts had already been started to form a body of scientific knowledge called as psychology of mathematics for instruction.

These experiments, which were going to be the basis of this field of knowledge, were "for the pupils who lack the ability, the interest, or the need for the usual courses in algebra and geometry", placed in the "applied mathematics" courses (Hawkins, 1940, p. 207). Although "considerable experimentation has been tried during the past fifteen years with the content of such a course or courses", numerous "practical problems" occurred in the operation of the plans (p. 206). The investigations, then, were stimulated by the generation of the scientifically valid and useful knowledge of "pedagogical implications" that is specific to mathematics (Wheeler, 1940, p. 30).

In one of the controlled experiments, for example, the effects of different methods such as dependencies (graphic or diagrammatical), the conventional-formula (four step), and the individual (absence of any formal method on the "ability to solve arithmetic problems" was investigated (Hanna, 1930, p. 442). In trying these three methods, the selection of a "reasonable sampling of children from urban life" (p. 445) had to consist of large numbers of students to generate scientifically valid knowledge. Nonetheless, the larger number students involved in the study, the greater irrelevant factors occurred such as "teacher ability". The irrelevancy of teacher ability was resolved with the attempt to equalize this factor through providing "definite teaching directions" (p. 446). The data gathered from these groups were studied with statistical techniques where the comparison was made the mean gains, measured with a "battery of standardized tests", in these three experimental methods. No matter which method demonstrated the larger gain, there was an implicit *commitment* to search for certain type of knowledge that was valid across time and space and applicable to other settings. The investigators were not just interested in any kind of answer to the specific questions; they were only interested in obtaining such answers on certain terms that have been set in advance where the end result is the "modern conception of the 'value neutrality' of scientific knowledge" (Danziger, 1990, p. 180).

Experimenting with various teaching methods did not only make an emphasis on "individual growth and development" (Hawkins, 1946), but also these experimental investigations were potent to "observe the rapid progress" over time through statistical calculation of the gains in median scores (Wheeler, 1940). The observed progress for this particular groups of students was considered to be applicable to other settings as well. The statistical tools enabled mathematics educators to generalize the conclusions driven from these experiments from the sample to the populations. As Desrosières (1990) argues, "using [statistical objectifications], social scientists have forged tools enabling them to transcend individual or conjunctural contingencies and to construct more general things that characterize for example the social group or the long term" (p. 196).

The "cumulative record" of the pupil (i.e. scores of intelligence tests, standardized achievement tests, school marks, teacher judgments, age, physical defects, special abilities and interests, vocational plans, economic status and occupation of parents and language spoken at home (Reeve, 1940, p. 128), quantified representation of these records, and controlling across different groups made the psychology of mathematics for teaching an empirical discipline that would provide a certain type of useful knowledge that could be transferred to different settings. Identification of psychological realities was understood in empirical regularities. The discipline of psychology of mathematics was established, which provided both "a reliable source of certainty" and "a practically useful knowledge" (Danziger, 1990, p. 193).

"The problem of the slow pupil" was to be solved not only by defining them in terms of particular populations and separating them their peers into different spaces, but also making them objects of research and intervention. Students were recognized and classified in terms of their measured "characteristics": The bright and slow pupils. They were separated into enclosed spaces, as a result, and this procedure was called either "ability grouping" or "tracking". Nonetheless, the political anxiety was more than these identification processes. Those who statistically deviated from the average score on the standardized tests had to be intervened. They became the object of research and these populations were acted upon through the controlled experiments. Finding the better or the best methods of teaching for these groups of students was to observe "a rapid progress" that was not only about an effort to increase their test scores but also a will to assimilate them into the fabric of "modern life" and "democracy". It was a simultaneous process of normalization of those who "deviated" from the norm and a technology of the self in the making of self-directed democratic citizens. The experimental apparatus in the discursive assemblage of school mathematics produce a knowledge that was "useful" and "scientifically valid", which assumed a mechanical, Newtonian worldview. That is, the knowledge formed in this apparatus was considered as having the capacity to be generalized across space and time, as truth that was built into representational premises.

3. Mathematical Learning Trajectories

In the contemporary mathematics education practices, we have started to see a widespread circulation of "learning trajectories" or "learning progressions" across the discursive assemblage of school mathematics. As I have discussed in the previous chapter, the shift from "discovering the mathematical world" to "mathematical modeling of the world" generates new forms of truth, objects of research and subjectivities. Practices have changed. Rather than teaching and learning the mathematical structures embedded in nature, Thompson (2011) calls for educators to be attentive to "how students conceive situations" with a "stance that quantities are in *minds*, not in the world" (p. 35, my italics). While this seems like an important shift in reasoning about the child and incorporates the constructivist movement in mathematics education (i.e. NCTM, 2000), it reinstalls the developmental continuum together with political anxieties about the nation, its growth and well-being of the population(s). However, the social and scientific practices, technologies and tactics that re-make this developmental machinery are not exactly the same.

In contemporary mathematics education practices, there is a growing movement to base standards, curriculum development and pedagogy on learning trajectories (Clements, 2011, p. 365). However, this is not only a scholarly desire to build a cognitive model of students' mathematical thinking, reasoning or ability, but also a strategy of change and development (Simon, et. al., 2010). The study of the cognitive development of the children through hypothetical learning trajectories might be conceived as one of the redemptive themes circulating in the 21st century mathematics education practices. Although the researchers have strongly claimed that constructivist approach reveals epistemological shifts in (mathematical) truth, certainty and subjectivity, in what follows, I will argue how these approaches become a re-iteration of historical questions of the developmental logic for the human race. This part of the chapter will not only describe the promises of these approaches and what makes them possible in particular time and space but also will consider the productive aspects of the this discursive assemblage in terms of what it does, what the limits are.

3.1. Re-forming the Mind

3.1.1. The Constructivist Move in Mathematics Education

In the constructivist movement in mathematics education, the change is mostly considered at the level of cognitive capacity of individuals. The investment of the human mind becomes one of the greatest hopes in the contemporary education reforms as reported in one of the documents of mathematics education, *Adding It Up*:

> Mathematics is one of humanity's great achievements. By enhancing the capabilities of the human mind, mathematics has facilitated the development of science, technology, engineering, business, and government. Mathematics is also an intellectual achievement of great sophistication and beauty that epitomizes the power of deductive reasoning. For people to participate fully in society, they must know basic mathematics. Citizens who cannot reason mathematically are cut off from whole realms of human endeavor. Innumeracy deprives them not only of opportunity but also of competence in everyday tasks" (National Research Council [NRC], 2001, p. 1).

Mathematics as human realization rather than discovery apparently cannot designate the highest stage of mathematical thinking in advance like the pre-post WWII reforms. Nonetheless, this "new" thinking of mathematics, which is a human product, entails different processes of ordering, comparing, classifying and normalizing the "mind" in the developmental ordering of the human nature and generates racializing and sexualizing discourses. Looking closely at this introductory quote, however, the two aspects of mathematics are revealed on the basis of the "realms of human endeavor": Knowing basic mathematics to participate fully in the society and the power of deductive reasoning as an intellectual achievement. Again, the human life where the abilities are unfolded into a series of stages is considered in a continuum (Baker, 1999). Nonetheless, the important point for us to consider is the fact that these reforms are made without an obvious articulation of "slow child" or an upper stage. What happened to the "slow child" with these new modes of scientific practice? How does this become possible? What makes it reasonable?

Do they suggest an infinitely many possibilities, which allow new ways of thinking, acting and being? Or are they just another boundary making process of the 21st century?

When mathematics has been considered as a product of human activity, this simultaneously has to reject the view that mathematical meaning is inherent in representations outside the human mind. The constructivist line of research does not assume the world or the universe inherently rational and mathematical independent from human experience. It develops a basic principle that, students produce the mathematical meanings through an interpretive activity that is either individual or collective or both (Cobb, Yackel & Wood, 1992, p. 2).

Prior to the constructivist move in mathematics education, as I have also described, the foundation for children's mathematical knowledge had been from the structures of mathematics that could be found in nature. Reflecting on this assumption, for example, Steffe and Kieren (1992) critiqued Jerome Bruner where he conceptualized the educational process as secured upon the structures of subject matter with a little reference to the human capacities for reason and logic (p. 713). The "necessity" to look beyond mathematical structures and to investigate what the mathematical knowledge of students serves the foundation of constructivist line of research.

Nonetheless, only the teaching and learning practices prior to this line of research and reforms were to be modified. That is, what these researchers called as "traditional" mathematics education was the thing to be changed and they criticized their object of change in several respects: The separation of the external structures of mathematics and the internal capacities of human has made the educational process a form of imposition from outside. Transmitting the finished mathematical structures to the students has created some problems of application of this knowledge in other settings since the learning process is not self-evident to the students.

Additionally, the predetermined stages are not flexible enough to accommodate instruction (Cobb, et. al., 1992). These problems need to be solved if one wants to advance the state of mathematics education. It is particularly this desire of moving forward and making progress in the mathematics education practices that drives the need of change. One of these redemptive moves was to initiate the constructivist approach that connects the children's experiences and the structures of mathematics. So, it was not completely eliminating the ontological existence of external structures either. As Cobb and his colleagues (1992) clearly admitted, while this new approach transcends the contradictions of the representational view, it offers another account of truth, certainty and subjectivity (p.3).

The tensions and concerns held by the mathematics educators, however, should be considered in the discursive assemblage of school mathematics. Was the constructivist movement only the effort of these researchers? Was it only about bringing new ideas of teaching and learning mathematics? Of course, it was not. What made these concerns and enabled the reform movement in mathematics education possible should be considered in a broader spectrum of social, scientific and political transformations rather than ideas of a few individuals. In what follows, I elaborate the historical intelligibilities that made the "constructivist" grid in mathematics education possible.

3.1.2. Preparing "the Nation" for the Changing World Order

One of the issues that became apparent during 1980s is the constitution of a system that has "dual mission" as stated in *Everybody Counts* (NRC, 1989), which was a report to the nation on the future of mathematics education in the United States. On the one hand, the aim of the mathematics education was to teach all students basic skills required for a lifetime of work in an industrial and agricultural economy and, on the other hand, to educate thoroughly a small elite who would go to college en route to professional careers (p. 11). Some students were considered as "the victims of crosscurrents in mathematics instruction, as advocates of one learning goal or another have attempted to control the mathematics to be taught and tested" (NRC, 2001, p. 11). This became the most serious and persistent problems in the field of mathematics education where the concern was not only about teaching and learning mathematics for "the victims" but also about both catching up and making the standards of "information age" with a desire to shape the future:

> Today's schools labor under the legacy of a structure designed for the industrial age misapplied to educate children for the information age. Not only in mathematics but [also] in every school subject, educators are faced with rising expectations for preparing the kind of work force the country will need in the future. Information-age technology will continue to grow in importance; pressed by rising international competition, industry will demand increased quality and increased productivity. The world of work in the twenty-first century will be less manual but more mental; less mechanical but more electronic; less routine but more verbal; and less static but more varied (NRC, 1989, p. 11).

Ability grouping has been regarded as harmful for the unity of the nation in terms of realizing the individual possibilities, hampering the national growth, and undermining the global leadership (NRC, 2001, p. 407). Sustaining the mathematical strength of the universities and research has become almost impossible due to two main reasons. First, very few students enter or complete their studies in mathematics related fields. Second, "many segments of the American population are underrepresented at every stage in the mathematics pipeline" (NRC, 1989, p. 17). This situation of urgency mobilized the educational researchers, policy makers and many other actors in the field to make a "change" in the mathematics education. "A common foundation of challenging mathematics" (NCTM, 2000, p. 368) for all students has become one of the salvation narratives of the 21st century mathematics education: "Mathematics is a realm no longer restricted to a select few. All young Americans must learn to think mathematically, and they must think mathematically to learn" (NRC, 2001, p. 1). So, the concern is not only about learning mathematics, but also about thinking mathematically as a precondition to learn.

These efforts, nonetheless, are not always about purely improving the mathematics education practices. The fear is also about the risk of standing behind as a nation, in terms of scientific advancement among others, which was inconsistent with the American narrative. This needs to be changed. All children have to climb "the every stage of mathematics pipeline", whatever the highest stage is. Similar fears also appear as an impetus in the urgent calls for educational reforms.

> Our Nation is at risk. Our once unchallenged preeminence in commerce, industry, science, and technological innovation is being overtaken by competitors throughout the world. This report is concerned with only one of the many causes and dimensions of the problem, but it is the one that undergirds American prosperity, security, and civility (National Commission on Excellence in Education [NCEE], 1983, p.1).

Although the language is about security, the narrative is coupled with civility and prosperity. School mathematics is no more for particular groups and there should not be separation for the "slow child", the curriculum needs to be planned for all members of this nation. Everyone needs to become responsible for the progress of his or her nation. Then, "enhancing the capabilities of human mind" or "investing the mathematical development of all" was not merely about teaching and learning mathematics but also concerns with the issues such as American prosperity, security and civility. The new mode of political rationality is to be achieved through individual actions as a rational citizen to ensure stability, growth and progress of the state instead of selected few or some kind of theological power. The elimination of the spiritual redemption and eternal salvation of the feudal order and the generation of "ethico-behavioral schema", in Wynter's (1995) words, was a form of enslavement of the irrational and sensory aspect of human nature (p. 17). While this is important to consider the corporeal regulations of the mathematically able bodies in the flow of everyday life, which I shall be attending to in the next chapter, for my purposes here I want to focus on what enabled the developmental reasoning about the mind. Although these discursive statements do never mention the word of "savage" as in the pre-post WWII reports and do not distinguish the "slow child", they do not avoid placing mathematics as "a hallmark of the educated person" (NRC, 2001, p. 15) and as a sign of "the survival of democracy in America" where the "gap" was widening between different racial and economic populations (NRC, 1983, p. 14). As an effect, the project of reforming the mind has again automated the civilizing machinery with different articulations, which embodied the developmental narratives. Now, those populations who need be civilized are also known thanks to statistical technologies that identify and name them. A colonizing continuum is re-inscribed through the inclusion of particular populations those were excluded before not "according to the students' perceived mathematical abilities" (NCTM, 2000, p. 368) as claimed, but as an effect of their *calculated* mathematical abilities.

3.1.3. Children are at risk!

Further distinctions for the kinds of people have emerged in the contemporary mathematics education practices. The language of risk has started to be utilized in educational reforms: "Wake up, America! *Your children are at risk*. Three of every four Americans stop studying mathematics before completing career or job prerequisites" (NRC, 1983, pp. 1-2, my italics). A shift occurred in the discursive practices of mathematics education that enable the switch from "slow pupil" to "children at risk". Nonetheless, this is not a merely change at the level of language or words but an indicator of the shift in practices and processes that make the object of governance possible and visible. How can we read these changes? One approach is to consider the emergence of contemporary risk-based security calculations (Amoore, 2011). That is, a specific form of abstraction derived through data or quantities of a situation and reveal associations to calculate uncertainty and to unfold future (p. 2). These risk-based calculations do not reveal strictly linear time and they are anticipatory. Children are not "slow" or "remedial" yet, but they are profiled as "at risk" that might fall into those categories with the generalizations of numerical relationships generated from national or international exam data, demographic or economic predictions. Data may not be collected from "children at risk", but they are the data derivatives of these statistical calculations. In the past, on the other hand, one had to utilize a range of introspective tools and devices to "see" the "slow pupil". The image of the "bright pupil" was known in advance due to the conceptualization of the highest stage of ability in terms of inherently rational and mathematical universe. However, contemporary practices, particularly the radical constructivist research and reform movements, do not prefigure the world as inherently mathematical yet they do anticipate the reforms and interventions, which would ultimately enhance the capabilities of mind because quantities are not in the world but product of mind (Thompson, 2011). So, the desire is to secure the uncertain futures and to prevent the remedial practices, like ability grouping, in advance.

The emphasis on risk reconfigures the future uncertainty as something that could be acted upon at present (Amoore, 2013), which is not only about a desire to shape the future but also making the future as a category that could be tamed by governing the present. In relation to this, the research and reform movements after 1980s have started to use the notion of risk in two respects. First, children are not being identified as "slow pupil" after the act of teaching but they are referred as "children at risk" in advance to secure the teaching that could result in remedial practices. When there are those children at risk in the classroom, teacher needs to act according to prevent such results. Second, children are not only those at risk, but also the nation is at risk (NCEE, 1983). The consequences of the prior practices such as having the dual mission for mathematics education has started to be "dangerous" in terms of national unity as well as sustaining the global leadership across the world. The political anxiety is not only about teaching and learning the subject matter, but also about the national and global concerns about the future of America. There is a risk of becoming "a divided nation" where the knowledge of mathematics supports a portion of the population; those who are productive and technologically powerful elite, while discourages others from these resources. The statement is cautionary but actionable: "Unless corrected, innumeracy and illiteracy will drive America apart" (NRC, 1989, p. 11). In addition to these internal concerns, there is a another political anxiety with the dissemination of international exams and comparison tests across the world: "Current mathematical achievement of U.S. students is nowhere near what is required to sustain our nation's leadership in a global technological society" (p. 4). Of course, the distress is not only about the mathematical achievement of students but also a matter of concern regarding the reconfiguration of American race and its prosperity in the globe through making kinds of people. Who would be the savior of the next generations in the world if the number of elite children in the higher tracks were declining day by day while the number of children was increasing in the lower tracks?

Consistent with this risk language, learning is also described in terms of chance. Students are "the victims", "deficient" in mathematical abilities or they have "limited understanding" since they are not given fair educational opportunities. The difference those who are achieving in the state, national or international assessments and those who are not is the fact that their learning opportunities are not maximized due to "traditional" approaches, and that is taken as the only problem to be solved.

State, national, and international assessments conducted over the past 30 years indicate that, although U.S. students may not fare badly when asked to perform straightforward computational procedures, they tend to have a limited understanding of basic mathematical concepts. They are also notably deficient in their ability to apply mathematical skills to solve even simple problems. Although performance in mathematics is generally low, there are signs from national assessments that it has been improving over the past decade. In a number of schools and states, students' mathematical performance is among the best in the world. The evidence suggests, however, that many students are still not being given the educational opportunities they need to achieve at high levels (NRC, 2001, p. 4).

The construction of difference is territorialized only within the learning environment defined by the scope of teaching and the opportunities the teacher can provide. The development of mathematical ability and the progress depend on this relationship between teacher and student: "How the students respond to the opportunities the teacher offers then shapes how the teacher sees their capacity and progress, as well as the tasks they are subsequently give" (p. 9). While this bare relationship appears ironic given the all rationales and discourses that make the constructivist movement in mathematics education possible, we should also note that what circulated among the constructivist line of movement is the introduction of new ways of knowing the mind without representational premises along with a critique of Cartesian epistemology. These opportunities of learning could only be understood if the teacher knew the change mechanisms of the cognition.

3.2. Studying How Reality is Constructed

3.2.1. Building the Cognitive Model of Mind

According to constructivist researchers in mathematics education, their effort has been a revolution in very radical forms, as they consider themselves challenging the long-held understandings circulating in science, epistemology and also mathematics education. To some extent, that was true. The interest is not about representing a reality or an ultimate product of mind. Rejecting the external representations (out of mind) and the view that the highest stage of mathematical ability could be found in nature, world or any space outside of the self, the research was interested in "studying the construction of the reality" following von Glasersfeld's work on epistemology rather than "studying the reality" (Steffe & Kieren, 1994, p. 721). This distinction in the unit of analysis and the move from studying "the reality" to studying "construction of the reality" have to do with the shifts in the social and natural sciences from product to process and from the predefined structures to the system that constitutes the reality (Heyck, 2015). As a result of reformulation of "mathematics" as a human product that models the world, which argued in the previous chapter, constructivist research has to look beyond the mathematical structures in the world in order to investigate children's mathematics. If children can mathematize a reality that is not inherently mathematical in model eliciting activities, their mathematics should go beyond what the nature offers.

The task or the "commitment" of the researcher, then, is to build a model of this constructive process and to understand this process as an outcome of individual-environment interaction (Steffe & Kieren, 1994, p. 722). This relates with the modeling practices of the reality as discussed in the previous chapter; however, this time, researchers are to *model* the child's mathematical thinking and reasoning. That is to say, "modeling" becomes a mode of reasoning that generates the scientific knowledge for the child's mathematical reasoning and thinking.

The ambition of modeling children's construction of mathematical schemes shares the aspirations of cybernetic researchers who want to "turn a world framed in terms of consciousness and liberal reason into one of control, communication and rationality" (Halpern, 2014, p. 146). This requires a new model of mind with "new" methodologies to account for and reorganizing the models of government and economy, what Orit Halpern refers as "algorithmic mind", which can be seen with the communicative mechanisms. As Steffe and Kieren (1994) similarly argue, in order to deal with the Cartesian epistemology, the researcher needs to change his or her understanding about the science: "No longer did it seem necessary to use the controlled experiment with its emphasis on statistical tests of null hypothesis and empirical generalization to claim that one was working scientifically" (p. 720).

These conditions yielded to the establishment of a community of researchers in mathematics education (Georgia Center for the Study of Learning and Teaching of Mathematics) working on the problems and limitations of the Cartesian self that is known by the external mathematical representations. According to them, previous scientific practices need to be replaced by "the experience of researcher", "conceptual analysis" and "social interaction" (Steffe & Kieran, 1994, p. 720). This social interaction in the scientific milieu, nonetheless, is not a process of imposition but a negotiation. It has to be a self-evidential experience for both researcher and children (Cobb, et. al., 1992). "Traditional" studies are inadequate to attend these aspects of the experiential reality. These researchers do not want to accept the assumptions of psychometrics and they have started to follow cybernetics as a scientific procedure (Steffe & Thompson, 2000). The assumptions and limitations of these practices of "normal sciences", in fact, have yielded to problematic consequences such as ability grouping, which is almost about to divide the nation. So, it is not only about bringing novel ideas to mathematics education but also a part of larger political distress.

The non-Cartesian deal of the mind, nonetheless, is more than a new articulation of science or an innocent concern for the unity of the nation. It is rather a project of "establish[ing] living models of students' mathematics" where these models are sensible only when conceptualizing human beings as "self-organizing" and "self-regulating" organisms (Steffe & Thompson, 2000, p. 287). Simply put, it is a project that reconfigures the liberal autonomous subject who continuously invests for his living capacity of mind in which the boundaries of growth cannot be set in advance by mathematical structures. Nonetheless, the rejection of the external mathematical representations is not a complete flight from the truth embedded in the inherently mathematical universe but the reformulation of what counts as mathematical truth within the experiential worlds of children. This new formulation is more visible in the following text:

> Mathematics does, however, provide one of the few disciplines in which the growing student can, by exercising only the power inherent in his or her own mind, reach conclusions with full assurance. More than most other school subjects, mathematics offers special opportunities for children to learn the power of thought as distinct from the power of authority. This is a very important lesson to learn, an essential step in the emergence of independent thinking (NRC, 1983, p. 4).

While the highest stage of mathematical thinking is not articulated, the mathematical thought is defined as inherent power embedded in the mind that is to be reached not testing with the mathematical truth but through calculating and planning the communications between the researchers and the child in the research setting (i.e. which problem/task to be given, what questions to be asked or how to prompt). Yes, this rejects the authority of mathematical structures, but reinserts another authority that is inherent to mind and has to make visible with scientific procedures that follow cybernetic methods. "This cybernetic reformulation of reason", writes Halpern (2014), "produced new forms of measurement and methods in the social and behavior sciences, encouraging a shift toward 'data-driven' research [...] as benchmark of truth, and as a moral and democratic virtue" (p. 148). In mathematics education research, the shift in the practices of rationality is from the null hypothesis of the controlled experiments to the hypothetical learning trajectories of the teaching experiments (Simon, 1995; Steffe & Thompson, 2010). Hypothesizing the possible trajectories brings the idea of future and re-inscribes the developmental reasoning about children's abilities.

The teaching experiments that focus on the practice of teaching displace the problems of knowledge possession with problems of agency under the shadow of learning trajectories. The question has become, what students do, rather than what students know, as the cybernetic research once asked, what machines might be built rather than what man he is (Halpern, 2014, p. 154). The interest is no longer in what students possess as mathematical knowledge but what students are mathematically able to do (or model) in a variety of situations. This only could be answered if the researcher is committed to build a model of students' construction of mathematical reality. Although mathematical ability is reformulated, is this a complete flight from the problems and limitations of representational structure of the world or alternative governing mechanisms that tame the agency by re-establishing the developmental logic and the differences between kinds of people with the mathematical learning trajectories?

3.2.2. Hypothetical Learning Trajectories: Making a Difference or Making of Differences?

Constructivist researchers in mathematics education are less interested in what the mind looks like; instead, what their line of research is swirling around the study of cognitive development of mind, more specifically the change mechanisms that yield to cognitive development utilizing learning trajectories/progressions as "exploratory tools" (Steffe & Thompson, 2000). At the same time, nonetheless, learning trajectories are not merely a research tool to investigate the cognitive development but a rationale for several policies and reform efforts in mathematics education (i.e. NGA & CCSSO, 2010; NCTM, 2014) and a base for mathematics education practices such as curriculum, standards and pedagogy (Clements, 2011). For example, the importance of this line of research is documented in one the specific reports on learning trajectories:

Since students' learning, and their ability to meet ambitious standards in high school, builds over time—and takes time—if they are to have a reasonable chance to make it, their progress along the path to meeting those standards really has to be monitored purposefully, and action has to be taken whenever it is clear that they are not making adequate progress (Daro, Mosher & Corcoran, 2011, p. 12).

According to this report, for instance, "an education gap" occurred between ambitious goals of reform and actual student mathematical thinking (p. 11). Since the proportion of the "disadvantaged" groups is increasing, not only investigating and knowing the learning trajectories but also understanding the change mechanisms of cognitive development would provide a useful tool and a promising approach to define the track of students. That is why, the trajectory along the path of cognitive development, or the mathematics pipeline as mentioned in the earlier reports, needs to be "monitored purposefully" and "action has to be taken" whenever the students are not making adequate progress. Nonetheless, this action is not about separation of students in different tracks into enclosed spaces but adapting the instruction so that they catch the track within a small amount of time. In relation to this, teaching gains an unquestionable importance while it was an "irrelevant factor" seventy years ago.

Given that the constructivist researchers are uneasy with the "traditional" methods to know the track of the children, a number of new technologies that are mostly adopted from the biological sciences are utilized. Before getting into this, first of all, let me note a few things about the mathematical truth that is re-inserted in this research milieu.

The research investigating cognitive development does not have a null hypothesis that is waiting to be proved or rejected. Hypothetical learning trajectories (HLT) do not contain fixed rules or static endpoints. Rather, they are conjectural; that is, they are consisting of hypothesis that can be modified, refined and evolved towards a firmer model of children's mathematics as the research progresses (Confrey & LaChance, 2000). Although HLTs are exploratory tools and elastic enough, which prevent researchers, teachers or teacher-researchers to set mathematical endpoints in advance, these conjectures comes from a "careful review and analysis of the literature" (p. 236). At this point, we need to ask the following questions: If we reject an external representation of mathematics, how does it become possible to anticipate a learning trajectory based on what is represented in the literature as children mathematics? If we think that we are interrogating the limitations and problems of Cartesian epistemology, how can we conjecture about children's mathematics that is grounded on what was previously explored in another space and time?

What we have, then, is a research field that is simultaneously contesting the metaphor of mind as mirror of (mathematical) nature and re-establishing new notions of mathematical truth:

> Instead, we suggest… viewing students as actively constructing mathematical ways of knowing that make it possible for them participate increasingly in taken-as-shared mathematical practices. From this perspective, mathematical truth is accounted for in terms of the taken-as-shared mathematical interpretations, meanings and practices institutionalized by wider society. The notion of mathematical truth is therefore dealt with paradigmatically (Cobb, et. al., 1992, p. 16).

As I have mentioned earlier, this shift has to with the cybernetic reformulation of the reason, rather than a question of the regime of truth. That is, the interest is no longer in what students possess or who they are; on the other hand, the concern has become about scrutinizing what they (mathematically) able to do, assuming that the investment in cognitive domain would construct humans as productive member of society. Of course, this has to do with making kinds of people. The continuous refinement of HLTs does not entail infinitely many possibilities of knowing and being; on the contrary, they re-inscribe the Cosmopolitan notions of self such as liberal autonomous human subject in unfinished forms (Popkewitz, 2008). That is, the "self" of the 21st century continuously is in need of educational investment in the process of making firmer models of cognition. The absence of mathematical endpoints or "highest stage of abstraction" is, in fact, bringing another layer to the process of making up people.

The practices, tactics, arrangements in the making up people, nonetheless, require new technologies. It has become arbitrary to establish the regime of truth(s) with the 20th century technologies such as statistical generalizations, psychometrics, and intelligence tests- what 21st century researchers refers as "traditional"- since they are not pragmatically applicable to the classroom settings. Also, what makes these practices unreasonable, as I have tried to articulate above, is a set of social, scientific and political transformations that we have still been making and experiencing. Of course, one cannot deny the influence of such tools on the contemporary practices. We have not completely left them behind. However, these macro-level causal arguments based on statistical measurements and tests are not commonsense anymore; at least for the ones who study children's mathematics. The scientific knowledge that is produced in this realm, nonetheless, has the capacity to become more authoritative and regulatory.

> We [constructivist researchers] formulate explanations, we make predictions, and we even manage to control certain events in the field of our experience which is the reality we live in. All this, and especially any attempt at management, involves what we call common sense and at times also scientific knowledge. The second is mostly held to be the more solid. We rely on it, and it allows us to do many quite marvelous things (von Glasersfeld, 1996, p. 116).

According to von Glasersfeld, the constructivist theory is not interested in whether there is an ontological reality or not; however, it has "the obligation to provide a model capable of showing how it comes about that" (p. 116). Put differently, the model has to produce a kind of knowledge explaining, "what has worked in the past and can be expected to work again" (p. 114). Then, first of all, "what has worked" needs to be recognized by some kind of observer in his experiential domain. Second, to reiterate what von Glasersfeld admits:

> Consequently human actions become goal-directed in that they tend to repeat likeable experiences and to avoid the ones that are disliked. The way they attempt to achieve this, is by assuming that there must be *regularities* or, to put it more ambitiously, that there is *some recognizable order* in the experiential world (p. 114, my italics).

These two principles of constructivist theory, although they do not signify an ontological reality waiting there to be discovered, "pertain to the ways and means the cognizing subject has conceptually evolved in order to fit into the world as he or she experiences it" (p. 114). But, who is this cognizing subject? What kind of world is he expected to fit in? Who can recognize "what has worked"? How does it become recognizable?

Constructivism, then, unproductively separates the questions of epistemology and ontology while offering paradoxical accounts for the scientific knowledge that it is articulated. Von Glasersfeld and many other researchers who could be located in mathematics education use "viability" in order not to get themselves caught in solipsism (i.e. mind alone creates the world). The necessity of being viable in dealing with experience does not require to match but it has to fit in a world where "there are or will be obstacles and constraints that interfere with, and obstruct, the organism's way of attaining the chosen goals" (p. 118). So, human beings are articulated as cognitive organisms or a living system whose learning processes have to be analyzed by utilizing microgenetic methods to understand the changes in mathematical thinking (Simon, et. al., 2010). While these methods put the validity of macro level statistical procedures that analyze conceptual changes into question, they re-allow the developmental reasoning that differentiates the human race possible through studying cognitive mechanisms that yield to a category of fitness to the experiential world. That is a question of the not the degree to which life on earth is adapted to the environment but a matter of social, political and historical anxiety investigating the kind of mechanism that allows to live.

Although it is claimed that the process is a kind of "natural" selection that occurs in the interaction of organisms in their experiential environments, I would like to read this process in relation to a concept that might be called as *economy of cognition*, a rationally organized and calculated milieu to order, classify, differentiate and normalize children's cognitive schemas. In the economy of cognition, change occurs at the cognitive level in the relative frequencies of the competing approaches as the new ones are introduced and the obsolete ones eliminated rather than a staircase metaphor that enforces the hierarchical stages in "traditional" studies (Siegler, 1996, p. 112). Although the process is informed by probabilities, this is not more than a reformulation of evolutionary logic in which new species emerge from the accumulation of adaptations. The absence of the highest stage, like the civilized man, merely allows the normalization mechanisms to operate in different ways such as competing with one another to survive without referencing oneself to the external representations. At the same time, the issue of viability re-inserts a regulatory dimension into this competition. When a variety of cognitive approaches are competing in a regulated field and "obsolete" ones have to be eliminated, the mechanism becomes fragmented and creates division within the biological continuum of the human race, what Foucault (2003) addressed by the mechanisms of "biopower". The whole complex of individual-environment interaction, as suggested by the constructivist researchers, becomes a war machine that targets "the elimination of the biological threat" (i.e. unfeasible cognitive schema) and "the improvement of the species" (i.e. building firmer models of students' cognition). Then, the elimination of the other, the right to kill, becomes possible but racist (p. 256). However, this racism is neither an ideology nor a mentality. The actual roots of it are deeper than this. As Foucault notes, "it is a technology of power… a mechanism that allows biopower to work" (p. 258). This war machine, this mechanism that allows the economy of cognition to operationalize, becomes one of the fundamental mechanisms in the field of mathematics education as well as in the normalizing society to secure the power relations.

We should also note that, "the political problem of the use of genetics arises in terms of the formation, growth, accumulation, and improvement of human capital" (Foucault, 2004, p. 228). Biopower, in fact, aims at the optimizing the conditions of living organisms. It has a positive function. Organizing and coordinating the whole complex of the sciences of mind throughout genetic features constitutes the formation of a biopolitical field that produces ability-machines applied to man-as-living-being. Nonetheless, making the life healthier and purer simultaneously produce an objective of "controlling the random element inherent in biological processes" (Foucault, 2003, p. 259) such as mathematical learning trajectories derived within and throughout genetic epistemology of the child. Then, while these practices have articulated new accounts for mathematical ability and aim at optimizing the ability-machines, a continuous configuration of an 'enemy' or an 'abject' that could pose a risk to stability of the economy of cognition is inevitable.

Put differently, in order to optimize the cognitive capacities, the risks need to be minimized or to be killed. Otherwise, the system becomes unstable and noisy.

These new articulations of mathematical ability, at the same time, put the scientific knowledge in a normative position concentrating on the "life", such as everyday practices and experiences, even though the starting point was to question the applicability of previous scientific methods in practical settings.

> Scientific knowledge, then, provides more or less reliable ways of dealing with experiences, the only reality we know; and dealing with experiences means to be more or less successful in the pursuit of goals. Scientific knowledge, then, is deemed more reliable than common-sense knowledge, not because it is built up differently, but because the way in which it is built up is explicit and repeatable (von Glasersfeld, 1996, p. 117).

Everyday practices and experiences of human beings, or basically the life of teachers and students, become a goal oriented, repeatable scientific activity. In the constructivist account of children's mathematics, the absence of external mathematical representations becomes paradoxical. This inconsistency is, however, productive in double sense. First, as the scientific mechanism regulates how to live rather than what to live, it might generate a potential to maneuver. Second, the "new" articulation of science according to Paul Valéry, as cited by von Glasersfeld (1996), as "collection of recipes and procedures that work always", reinserts a "faith" that "rests entirely on the certainty of reproducing or seeing again a certain phenomenon by means of certain well defined acts" (p. 117). This brings the new agents of change and objects of research: Teachers. That is, projecting the scientific mechanisms in the experiential realm of children is the new salvation theme of the contemporary practices in mathematics education. The "faith" in science is not more than a new enunciation of Judeo-Christian religious images. However, these articulations are productive. While the saviors are teachers who can act accordingly and can execute those well-defined scientific mechanisms, at the same time, they become the objects of research, which examines these desired teaching practices.

3.3. Teaching as an "Anticipatory Thought Experiment"

The purpose of the research on learning trajectories is to investigate the mechanisms that yield to cognitive development in a precise manner to offer an account for desired teaching practices. Although the generated body of scientific knowledge does not provide an exhaustive list for what to do in teaching, hypothetical learning trajectories generate a normative aspect for teaching in the regulation of teaching practices. In one of the most cited pieces of Simon (1995) that is found a seminal work on HLTs, he mentions that the use of the term, "hypothetical learning trajectory," is "to refer to the teacher's prediction as to the path by which learning might proceed. It is hypothetical because the actual learning trajectory is not knowable in advance. It characterizes an expected tendency" (p. 135). The discourse of "being unknowable in advance" is crucial since it characterizes a teaching body who can predict the "student at risk" in relation to HLTs, who can act based on these probabilities not yet realized. This cannot be a dominant force. On the contrary, it controls and regulates the territory of teaching by making the teacher someone who is active in this process and who is reflective about what comes or what yet to come. Meanwhile, the teacher is not the only active agent in this process. Since HLTs are not independent from children's mathematics and their experiential realities, although this has a lot of paradoxes and limitations as I have discussed above, children are not a passive recipient of knowledge but active members of this teaching-learning terrain. Then, a relationship occurs between teacher and student mediated by HLTs.

> The development of a hypothetical learning process and the development of the learning activities have a symbiotic relationship; the generation of ideas for learning activities is dependent on the teacher's hypotheses about the development of students' thinking and learning; further generation of hypotheses of student conceptual development depends on the nature of anticipated activities (Simon, 1995, p. 136).

The hypotheses made by teachers are different than the null hypotheses of controlled experiments not only because of the fact that teachers are active in these process but also the dependency on the anticipatory logic. That is, the imagined future is brought to the present to regulate the teaching-learning terrain. But, I should note that "the imagined" is not someone's dream or an individual's psychic phantasy. Instead, it is calculated through the probabilities of various cognitive mechanisms in terms of viability or workability as I have detailed in the previous section in the face of some fears and anxieties, which do not only entail teaching and learning mathematics but also embodied a broader spectrum of political distress. The highest stage of abstraction is unknown in advance for sure to test whether the student attains this stage or not. However, the *hypothetical* learning trajectories with an emphasis on probabilities in the research realm and with an emphasis on the risk in the contemporary reform and policy calls become instrumental in making instructional decisions. "What matters is not the accuracy gleaned from large volumes of data, analyzed and statistically assessed", writes Amoore (2013), "but the intelligibility of the derivative as an instrument, its precision as basis for decision" (p. 67). So, mathematical learning trajectories, a body of scientific literature which are the derivatives of children's mathematics, function as a rationale for organizing the teaching and learning processes. Teaching becomes a techno-scientific intervention. Since trajectories "involve hypotheses about order and nature of growth", they are regarded as useful tools for teachers (Daro, et. al., 2011, pp. 12-13). It might be true that they serve as basis in the making of standards; however, it is important to look at the operation of these in teaching and learning spaces. Mathematical learning trajectories are not fixed entities; they need constant modification not only prior to instruction but also during the instructional time to ground a basis for the ongoing teacher decision-making. Put simply, the teacher's role is defined as "decision-maker" and teaching is "professionalized" as such. The modification of the hypothetical learning trajectory is not something that only occurs during planning between classes. The teacher is continually engaged in adjusting the learning trajectory that he has hypothesized to better reflect his enhanced knowledge (Simon, 1995, p. 138).

In some of the venues, the continual adjustment is elaborated as "adaptive instruction" where instruction has to adapt students "to try to get-or keep-them on track to success" (Daro, et. al., 2011, p. 15). Then, "building a cognitive model of students' learning" also serves to identify the track of students and to adapt them simultaneously in the moment of teaching, where the accumulation of adaptations yields to a "firmer models of student's mental activity" (Steffe & Thompson, 2000). Nonetheless, this adaptation cannot be arbitrary or be left to chance. On the contrary, these adaptations or decisions, which teachers make, need to be grounded on the "scientific knowledge" derived from the teaching experiments and serve to construct "scientific knowledge" while teacher is provided with an "opportunity" to adapt these knowledge in his or her own practices.

The instructional activities used in, for example, a teaching experiment, illustrate one concrete enactment of the sequence. When a sequence is justified solely in terms of traditional experimental data, teachers know that the sequence proved effective elsewhere, but they do not have the opportunity to develop an understanding that would enable them to adapt the sequence to their own situations. In contrast, the type of justification derived from an analysis of classroom mathematical practices offers the possibility that teachers will be able to adapt, test, and modify the sequence in their own classrooms (Cobb, 1999, pp. 31-32)

Comparing with the "traditional" data, it is not possible to deny the intent of prevention the particularities, such as marking the child as "slow pupil". Nevertheless, this does not mean that contemporary teaching and research practices do not act upon the children through the very modification of the calculation of risk in our present times. As Amoore (2013) argues,

> The contemporary risk calculus functions through the arraying of possibilities such that they can be acted upon. The significance of the array is that it allows for multiple possible sequences of events to be held together within a single purview. In this sense the array, as a spatial calculus, begins to occlude the series or seriality of prudential risk techniques (p. 69).

The prevention of singularities as a product of disciplinary techniques has transmuted into another form of enclosing by the very "scientific" calculation of the space. That is, while multiple possible trajectories in the teaching and learning terrain are allowed to co-exist, the desire to reach the "firmer models of students' mental activity" by the very act of teaching operates as an exclusionary matrix in the face of risk and security. The process of formation of this matrix, as mentioned in the previous section, is also informed by the biological sciences and by the probabilistic accounts that make the inclusion and exclusion possible through an economy of cognition that is based on the relative frequencies of the competing approaches and organized within and through several other tactics and technologies. An evolutionary logic is inserted in teaching practices in the name of science.

In fact, as Clements (2011) clearly states, "the power and uniqueness of the learning trajectories construct stems from the inextricable interconnection between [psychological developmental progressions] and [instructional sequence]" (p. 365). Although nobody can deny the relations between teaching and learning, the construction of mathematical learning trajectories, which is a process named as "anticipatory teaching experiment" (Cobb, 1999, p. 6), reinscribe the developmental and evolutionary reasoning about the human race by envisioning the instructional sequence and by envisioning how students' mathematical learning might proceed.

> It is, however, feasible to view a hypothetical learning trajectory as consisting of conjectures about the collective mathematical development of the classroom community. This proposal, in turn, indicates the need for a theoretical construct that allows us to talk explicitly about collective mathematical development… Described in these terms, a learning trajectory then consists of an envisioned sequence of classroom mathematical practices together with conjectures about the means of supporting their evolution from prior practices (Cobb, 1999, p. 9).

The "ambitious goal" of teaching, then, is articulated in two points: First, teacher needs to "understand what the 'track' is, in some detail"; second, teacher needs to "know what is likely to help keep a student moving forward on it, or to get him or her back on it, if they are having problems" (Daro, et. al., 2011, p. 55). In this account, mathematical learning trajectories are used as "a codified body of knowledge that provides [teachers] with pretty clear basic ideas of what to do in response to the typical situations" (p. 16). In a similar vein, Sztajn and her colleagues (2012) argue that professional knowledge of content and teaching needs to be reinterpreted based on learning trajectories in order to "support learners' cognitive development through progressively more sophisticated levels" (p. 149).

Then, what is articulated as ambitious teaching, under the shadow of professionalization attempts, is not more than a technology that reinscribes the hierarchical scale of cognitive development. If one historically encounters these discourses by bringing them together, mathematical learning trajectories emerge as techno-scientific practice that re-makes the developmental machinery circulating in the contemporary mathematics education practices while the articulation of (mathematical) ability is changed. A conceptual mutation occurred at the level of human cognitive capacity, which could be still scaled in a continuum of learning trajectories and could be refined both against and through the real world. Rejecting the scientific and transculturally verifiable image of the world brings the range of human heredity variations, which enabled pure biologization of cultural modes of being and mapped onto a linear evolutionary scale (Wynter, 1995, pp. 38-39). While the constructivist researchers claim that these practices are territorialized in the mind, ongoing tracking of the children's cognitive capacities in terms of "viability" in their everyday life and experiences makes the human life a new target. And, this is inseparable from the population dynamics as it is generative in terms of constructing differences between kinds of people.

4. Conclusion

In the previous chapter, I have discussed the move from "discovering the mathematical world" to "mathematical modeling of the world". Although articulated differently, the enactments of these two produce a culturally intelligible domain in the making of mathematical world and self. This chapter brings another layer to this space with the rationalities of the sciences of mind. That is, those who can tell the truth as moral subjects by means of the operation of communicatory technologies such as such as "mathematical precision", "accuracy" or "numerical validity" is now distinguished in a continuum in which the other end consists of those who do not have those "capabilities" yet. Then, not-yet-developed minds have become the object of reform and intervention. As Baker (2013) notes, mind has become a "potential site of unity" to locate the old assumptions about the onto-epistemological hierarchy into new rationales and tactics.

Although we have seen a departure from the practices in 1930s-40s with the constructivist move in mathematics education practices, seemingly new methods and technologies do nothing more than re-inscribe the old assumptions that put human race in a continuum and create differential effects. Although researchers argue that this is a shift in epistemology (Steffe & Kieren, 1994) or a new understanding of mind from a non-representational perspective (Cobb, et. al., 1992), the analysis makes visible how the historical questions in the developmental logic are reformulated. The uncertainty of future is taken into account; however, with the contemporary calculation of risk and anticipatory logics, the developmental machinery is put into operation in an economy of cognition to act upon people and populations. The crucial thing to interrogate is not whether there should be developmental machinery or not; on the contrary, the task is to explore the discursive practices that make it possible and seemingly natural.

Chapter V

Disentangling the "Body": From Possessing Mathematics to Doing Mathematics

1. Introduction

The historical analysis of the two moments in the mathematics education practices makes the corporeal regulations visible while it reveals a transmutation from "possessing mathematics" to "doing mathematics" in the everyday bodily enactments. In this chapter, I study how "mathematics" is becoming an essential part of life in four lines of argument that show parallelism in the practice of math-for-all with the aim of "progress" and "development". For each moment I first explore how "mathematics" becomes an essential necessity to pursue a life as a human and as a citizen of the nation. The second point is about how "democracy" becomes operationalized as an apparatus for the masses to embody bright mathematical futures. Third, these futures are seen through psychologization processes of "others" who do not fit into those futures, yet they need to be identified, reformed and corrected. The last point is about the planning processes to intervene in order for those identified as "other" or as "diverse" populations to be integrated to the collective whole. Once it was to track those children into low level "social mathematics" courses, the strategy has moved from curricular planning towards planning of effective teachers who are to embody a *model* of an effective pedagogy, which is ordered by the body of knowledge in the mathematics education field.

2. Possessing Mathematics for Life

One of the discourses permeating the pre-post war discursive assemblage of school mathematics was the "mathematical world". In Chapter 3, it was the practice of "discovering the mathematical world" that was put in action to legitimize studying "mathematics" in schools.

However, these practices, which had a close relationship with the Enlightenment reason and rationality, were to make particular kinds of people, with an image of rational man, and were to become a mechanism to distinguish children into different tracks. While there were developmental narratives as of one the rationalities that made possible and intelligible this separation, if one wanted to legitimize mathematical study for all, the lower tracks had to be made desirable for the "rest" of the school subjects.

Enrolment in the lower tracks was not meant to be a punishment for the "slow pupils"; the plan was to rescue them. All children had to "possess" some kind of knowledge to meet the "needs" of everyday life, even though some of them were not "mature" enough to discover the mathematical world. Then, taking the "mathematical world" for granted was productive to make up people. While there was an aim to cultivate a "man of culture" who was mature enough to contribute to the technological and scientific orientation of society, an effective program was put into operation for the *all* who had "mathematical needs" to maintain their life. It was the "dual responsibility" of school mathematics:

> (1) To provide sound mathematical training for our future leaders of science, mathematics, and other learned fields, and (2) to insure mathematical competence for the ordinary affairs of life to the extent that this can be done for all citizens as a part of a general education appropriate for the major fraction of the high school population (NCTM, 1945, p. 195).

2.1. Meeting the "Mathematical Needs" of Everyday Life

An accumulated linear account of mathematical knowledge was adopted in the making a program of "mathematics for all", where "much of mathematical progress" has been articulated as "a direct response to the immediate practical needs of everyday life of the community" (PEA, 1940, p. 242). While the narratives of "math for all" entailed a general purpose of mathematics for social and scientific progress and development, they were to order, cultivate, direct inner qualities and personal characteristics of the child as an object of governance and a future citizen: "The major role of mathematics in developing desired characteristics of personality lies in the contribution it can make to growth in the abilities involved in reflective thinking or problem solving" (PEA, 1940, p. 59).

The reason for "math for all" was not only concerned with mathematics as content, but more about the managing and controlling human conduct in daily life through the "development of desired characteristics of personality". Possessing mathematical knowledge was to meet the "needs" of the individual in the major aspects of living such as personal life, immediate personal-social relationships, and economic relationships (PEA, 1940, p. 75). Nonetheless, the principles of participation, communication and maintaining the social stability were ordered and normalized based on the problematic of human needs. How does mathematics come to be possible in normalizing and the directing the inner qualities and personal characteristics of the child as a future citizen? How could the "inevitable" questions for human beings only be answered through *counting* and *measuring* (NCTM, 1940, p. 2, italics original)? How did counting and measurement become possible to think as human needs?

2.1.1. Historical Emergence of Mathematical Needs

One way to think of the practices related to measurement and counting as "mathematical needs" of humans in daily life is to historicize the problematic of needs as a process of making self-directed citizens to make the modern nation possible. In eighteenth century France, for instance, there were approximately 700 or 800 differently named measures and untold units of different sizes used to measure the same thing (Heilbron, 1990). While non-state forms of measurement grew from the logic of local practice, a lot of micro politics were occurring in early modern Europe. People were playing with the size of baskets in the exchange and trades of goods to maintain power relations (Scott, 1998). These “random” practices had to be changed to preserve the social order and stability. However, this was more than a way of resolving these confusions, arbitrary exchanges and barbaric practices but also about making self-governed individuals for the modern society.

The local measurement practices, also, made it difficult to monitor markets, to compare prices for basic commodities and to regulate food supplies effectively. The growth of market exchange, Enlightenment favor of single standard of measurement, and the French revolution combined with Napoleonic state building made the “metrical revolution” possible (Scott, 1998). There were requirements for the new system of measures so it would not be resting on arbitrary units, not offer incentives to cheaters, be easily reproducible, be rational so that it can become universal and it had to be simple. The new system of measurement should be easily available to the ones “with the lowest and humblest capacity”. As this was a process of making the equal and free citizen, everyone should be able to confirm by himself about the correctness of all these transactions (Heilbron, 1990, pp. 208-209). Permitting the comparisons to be easier and more manageable, these common measures were to make not only the commerce and industry more efficient and productive, but also promote rational citizenry and equal citizenship (Scott, 1998, pp. 32-33).

The making of citizens in the modern secular societies requires inventing different technologies rather than using brute force to ensure the liberty and equality of the modern citizen.

To maintain social stability with self-governed individuals, the principles of participation and communication in multiple spheres of life need to be arranged and orderly planned. As the operation of civil society could depend on self-rule rather than ruling by coercion, liberal theorists started to reconsider, as Poovey (1998) argues, the kind of knowledge useful for the government. That was less about measuring productivity, but more concerned with the administration of self-rule in a market economy through understanding of human motivations such as the desire to consume. As a result, the useful knowledge that was essential to liberal management of individuals was not "the kind cultivated by moral philosophers" but it had to be established with the scientific method for "an account of subjectivity that helped explain desire, propensities, and aversions as being universal to humans as a group" (p. 147). That is, the issue was not only about how to order individuals to pay their taxes on time or to consume goods in the market, but about making them to see a "reason" to pay taxes. In this sense, standardization of measures was also serving individual happiness and well-being since humans did not need to consult anyone but mathematical calculations about their exchanges, prosperity and expenses.

In making a program for rationalization of the economy and society, a new and "scientific" knowledge of man was necessitated not with an intention of making "the people" as merely objects of the market but making the exercise of power happen within the sphere of self-rule. While "the man" was not merely the cog in the machine in the first instance, analyzing the subjectivity of human behavior in the economic system made the man as the object of planning. Foucault (1970) argues,

> 19th century economics will be referred to anthropology as to a discourse on man's natural finitude. By this very fact, need and desire withdraw towards the subjective sphere-that sphere which, in the same period, is becoming and objects of psychology (p. 257).

Then, there remained a double image of mankind to live in the face of death and produce negativities as well as positivities: A livable or an unlivable life. That is, the analysis of the "natural finitude" was to constitute the man's own humanity and was to figure out what he "needs" from outside of the self. This kind of man is "not the human being who represents his own needs to himself, and the objects capable of satisfying them", but he is "the human being who spends, wears out, and wastes his life in evading the imminence of death" (Foucault, 1970, p. 257). Considering the linear evolutionary account of Time and History, relatedly "accumulation" of knowledge, this was not simple like: You need food; otherwise you will die. On the contrary, the struggle to live in the face of death generated a scientific discourse about the human essence where the analysis has to do with studying the way human capacity was formed and accumulated where the value was analyzed in terms of *need*.

While to live has become an effort to maximize human capacity, the scientific discourse about human essence simultaneously constituted "an empirico-transcendental doublet which was called *man*" (Foucault, 1970, p. 319). The whole corpus of knowledge that made possible the rationalization of economy and society, utilizing the human's natural finitude in the name of "needs", produced a regime of truth making and dividing kinds of people. This was also the exercise of power. "Knowledge of man," writes Foucault, "is always linked, even in its vaguest form, to ethics or politics; more fundamentally, modern thought is advancing towards that region where man's Other must become the Same as himself" (p. 328).

During pre-post WWII, then, the widespread circulation of "mathematical needs" is important to take into consideration in problematizing the "reason" of "math-for-all". The discourse entailing that everybody had mathematical needs simultaneously became a dividing practice. Put differently, mathematical needs were not only a technology of self but also a technology of making of difference. Several reports, for example, were consisting of "ideas and definite suggestions from which a sensible report may later stem that will provide adequate training in mathematics for all students in our schools-each *according to his need*" (NCTM, 1944, p. 226, my italics). While the discourse of "all need math" were taken for granted, it produced another layer to distinguish between kinds of people:

> We should differentiate on the basis of needs, without stigmatizing any group, and we should provide new and better courses for a high fraction of the schools population whose *mathematical needs* are not well met in the traditional sequential courses (p. 228, my italics).

What were those mathematical needs? Did everyone have mathematical needs? If yes, why were there "different" mathematical needs that required differentiating? Was there something "inherent" in "mathematics" that satisfied those (mathematical) needs?

2.1.2. Socio-spatial Configuration of "Mathematical" Life in 1930s-40s

Given the fragile position of mathematics in the school curricula at the beginning of the 20th century, the "social aim" of mathematics was emphasized in the reform efforts to make studying mathematics worthwhile for everyone not just particular groups of elites. Embodiment of mathematical modes of knowing was a re-enactment of the historical concern of governing people in their everyday lives in modern societies. Nonetheless, when life is taken as a fundamental target in the management and planning, new forms of objects and methods are produced (Foucault, 1970).

> Their [slow pupils] reading vocabularies can be sufficiently developed for them to appreciate and understand the quantitative expressions commonly found in newspapers and magazines. If the school is equipped with a good mathematical display and the material in it is properly explained, the backward pupil gains a greater insight into role of mathematics in civilization than sometimes suspected (NCTM, 1940, p. 143).

One way to think of mathematization of life during the pre-post WWII period was the desire to strengthening the bonds with the state during or after the war. The increasing search for useful knowledge for the socio-spatial management of individuals became a crucial question for governments. Mathematical modes of acting and participating in everyday life became important not only to maintain the social stability but also to make particular kinds of people: "Mathematical study is desirable not only because it is useful but because it helps in a unique way toward intelligent adjustment in the present-day world" (NCTM, 1940, p. 141). In planning for the post war society and school mathematics, "intelligent dealing" with the real life issues became essential for functional competence and preparing youth for an adult life:

> Does he have a basis for dealing intelligently with the main problems of the consumer; e.g., the cost of borrowing money, insurance to secure adequate protection against the numerous hazards of life, the wise management of money, and buying with a given income so as to get good values as regards both quantity and quality (NCTM, 1945, p. 198)?

Although the above text implicated possession of mathematics for a personal life, the report was also concerned with the management of family and community to regulate the economic life of the masses:

> Does he have the information useful in personal affairs, home, and community; e.g., planned spending, the argument for thrift, understanding necessary dealing with a bank, and keeping an expense account… making change, and the arithmetic that illustrates the most common problems of communications, travel, and transportation (NCTM, 1945, p. 198)?

The "reason" of "math for all" was not only concerned about mathematics but also about ordering (im)proper modes of life for the self, family and community (Popkewitz, 2008). The position of the reforms, briefly stated, was principally this: "A mathematics curriculum may be built by locating and studying concrete problem situations which arise in connection with meeting needs in the basic aspects of living" (NCTM, 1940, p. 73). "Math for all" was, then, embedded in the larger social and historical transformations that entailed an increased demand for intelligent and efficient citizens in planning of the postwar society. Put differently, "math for all" was productive in making the self and society by standardizing particular modes of life. Nonetheless, standardization is not just a matter of the imposition of a system of bureaucratic regulation but a condition for interaction in diversified societies with an expanded division of labor and common means of trading that would provide a "translatability" that enabled coordination between different kinds (Porter, 1995). As suggested in the introduction of standardized measures in the 18th century, it was not only the order of those who were the rulers but also a condition that was produced following the changes in governmental practices. However, given the absence of moral authorities or sovereign rulers, a technology has to be invented to assure trust between people and to eliminate subjectivity in communicative relations. That is, a type of objectivity was constituted. Life, which emerged out of, and was superimposed upon the contentious and the uncertain, became a network of the apparently precise, specific and quantitative (Rose, 1999).

Possessing the skills that made the objective knowledge possible such as precision and accuracy, which were not specific to mathematics, were regarded as necessities to be acquired by students. For example, Shamhart (1942) argued "preciseness and accuracy are basic necessities to everyone in the commercial world of today and in any phase of life" while explaining the aims of school mathematics curriculum. Possessing these qualities became a regulatory mechanism where the emphasis was on the adjustment of those who were located in the premature mathematical spaces (Douglas, 1943). That is, it was not the "mathematics" but the specific skills such as precision and accuracy in the lower tracks fundamentally had an "enabling" function for those children excluded from higher tracks.

Embodiment of mathematical modes of living became the motor of development and progress, which maintains the historical concern of governing people in modern nations and planning the society. The "civilized" standards of the rules of conduct assembled with the Enlightenment notions of Cosmopolitanism, such as being truly universal and a model to the world, which has become one of the greatest hopes of the American nation to produce future Cosmopolitan citizens (Popkewitz, 2008). Besides these hopes, the design of the child as future citizen incorporates fears of darkness and backwardness. The double gesture embedded in this assemblage generated both a regulatory mechanism that fabricate kinds of people and an exclusionary matrix making the abjection of those who might act outside of these regulated spaces possible.

While the "mathematical" competencies such as precision or accuracy were part of the organization and the exercise of power in the field of mathematics education to generate particular kinds of people and modes of life, they simultaneously re-territorialized into the inclusive statements such as "math for all" to meet the mathematical needs of "all" to maintain everyday life. It was to survive in a life historically constituted as "mathematical". Then, we have this paradoxical mechanism undergirded into the reason of "math for all", which, in fact, gives us an impetus to recognize *politics of life* circulating in this field. The enabling function of particular living modes made possible a mechanism that targeted not only the lives of individuals but also the collective lives that merged with their cultural environments as an exercise of power in managing the self and the society.

2.2. Democracy in and through "Math-for-All"

2.2.1. Establishing "Mathematical" Common Ground for a "Democratic" Country

There is also another practice operating in the "reason" of "math for all" that goes beyond teaching and learning mathematics but entails the idea of living together in a modern society. Educating "the neglected groups" became a moral responsibility to ensure the collective belonging to the community and the nation. This had to do with the mutation of the way of understanding the collective existence of people following the collapse of sovereign empires (Rose, 1999). More specifically, in the American Enlightenment, writes Wood (1991),

> ...once men came to believe that they could control their environment and educate the vulgar and lowly to become something other than what the traditional monarchical society

> had presumed they were destined to be, then they began to explain their sense of moral responsibility for the vice and ignorance they saw in others and to experience feelings of common humanity with them (p. 237).

It became clear that the people with particular characteristics could integrate to the whole through a certain moral order to establish a common humanity and maintain the "democracy" in liberal states: "The maintenance of democracy today is predicated upon the ability of large numbers of people to think clearly about problems that are essentially statistical in character" (PEA, 1940, p. 66). This "development" of the "ability" to solve problems was considered as important since it provided "a common ground for cooperative development of an essential characteristics of personalities equipped to function creatively in a democracy" (p. 63).

While the establishment of "a common ground" by mathematical means was a hope, there were two different fears operating and creating political anxieties with regard to self and society. First, apparently, was the destruction of this common ground and pulling apart the collective existence of citizens. Second was the fear of anti-democratic societies.

> Democracy rests upon faith in the intelligence of common men. Once this faith is destroyed, the alternative is some form of dictatorship, which means the destruction of democracy itself. The schools have never yet consciously and deliberately organized their programs in a way to encourage and promote intelligent action on the part of all people (PEA, 1940, p. 30).

In the re-formation of the nation, following the war, strengthening the collective bonds and cultivating pupils as "loyal Americans" were to resolve the internal and external concerns of the country. This had to do with the intelligence, the moral stamina and the heroic stalwartness of youth of the day and made possible a colonial continuum of the human race. In effect, the faith in the intelligence of common men produced a redemptive discourse for those who needed to be democratized and generated an ethico-political rationality for "math for all" based upon the legitimacy of those holding the political authority and those subject to it (Rose, 1999).

The desire to establish a common ground among people through possessing mathematics was more about ensuring the collective existence of people, whose "differences" were taken for granted from the beginning, in modern societies in the name of "democracy". This was the enunciation of the desire for a society that could be united even in its diversity.

> Because our Democracy is made up of the various components: different 'races,' cultures, mores, languages, dialects, religions, nationalities -all welded into one complete whole- a whole greater than any of its parts; a whole for which men give their lives in battle that Democracy may be preserved (Mansfield, 1944, p. 250).

Everyone needed to become responsible for the progress of his or her nation. It was a battle on the home front for the national well-being and self-representation. However, this was productive in other respects, too. They were about making kinds of people "who embodied the will of the nation and its images of progress" (Popkewitz, 2004, p. 7).

These practices were to regulate and calculate the freedom of citizens. The establishment of mathematical common ground was an effort to standardize communication, participation and social relationships not only in school but also in the life. It was to calculate boundaries for what could be said. Nonetheless, these practices were far from forcing people to build and act in the common ground but more related with the cultivation of particular characteristics entailing a shared will to do so. Then, the desire of "math for all" was more about making particular kinds of people as "democratic" citizens with "democratic" methods. "Democracy" was a strategy of governing of the self and the masses (Cruikshank, 1999). The embodiment of the common (mathematical) ground organized cultural inscriptions to belong to the collective all.

2.2.2. Mathematical Standards of Life in a "Democracy" and "The Social Question"

The will in the "math for all" was not only about establishment of a common ground to make the nation with equal and free citizens but also an inscription of corporeal regularities in the everyday life of people. The quantitative organization of life with this particular way of thinking was to establish cultural standards of living in these calculable spaces. The absence of a dictator did not naturally result in making democratic citizens. On the contrary, it was this particular effort to change the ways of living to constitute citizen capable of fashioning a self that was governable (Cruikshank, 1999, p. 124). Then, "math for all" was not a merely rhetoric moving along the discursive assemblage of school mathematics, but a discursive practice that made up people.

> The underlying philosophy that "all men are created free and equal" may only come to have meaning, to persons who have never known democracy, when they learn that democracy offers an opportunity to eat balanced meals, enjoy suitable clothing, earn higher wages, elect representatives to governmental offices, etc. (Mansfield, 1944, p. 250).

"Math for all" was not only a simple tool to preserve the "democracy" but also a practice that governed people in their daily life. Were these characteristics, such as "eating balanced meals" or "enjoying suitable clothing", "participating in the economic activities" and "voting", constituted as the standards of life in a "democratic" country? Regardless of the answer, it was not the mathematical capabilities but a set of living habits that made the democratic citizens.

In the face of dictators who had mobilized the masses into the war, the fear held in the liberal democracies, such as the United States, made the authorities to be concerned about both maintaining and policing the national will, unity and purpose (Katznelson, 2013). So, the "democracy" entailed as opposed to the dictatorships was never about total freedom and liberation. On the contrary, "democracy" became the very apparatus of the state and a technology of governing the masses. This apparatus of democracy was one of the tactics that legitimize the "math for all" and regulate the masses. "In a democracy", reports the Committee on the Function of Mathematics in General Education, "where each person is expected to take part in policy-making and to direct his own life, the disposition and ability to analyze problem situations is peculiarly necessary" (1940, p. 39). Although this was not a dictatorial regime as feared at that time, it was a regime of mathematical truths that shaped and fashioned the life of its citizens. These "mathematical truths" to govern life of the citizens made possible a mechanism that ordered, differentiated, classified and normalized particular kinds. That is, domesticating the eating, clothing or consuming habits produced *(un)livable lives*. "Math for all" was nothing more than the re-inscription of the civilizing mission of "mathematics", as actualized by socio-spatial configuration of "mathematical truths".

> We desire more adequate food, clothing and shelter; we wish to reduce economic insecurity; we are determined to conserve our natural resources; we seek more whole some uses of leisure; we need more efficiency in government. A better civilization is to be created (NCTM, 1944, p. 227).

The re-inscription of the scientific thinking about nature onto social spaces entailed a similar mechanism that standardized its kinds. The identification processes of humans, as discussed earlier, however, were possible with a developmental logic about the humans, which, in fact, re-establish colonial thought. In this conjuncture of "math for all", the consistent use of adult life such as paying taxes or getting an insurance, and identification of those who "need" mathematics though categories like "adolescence" or "young children" were part of the grid that made the intervention for those who had not embodied the mathematical standards of life yet. In explaining why the mathematics was the key to democracy, the following was stated:

> Adolescents not only need help in reformulating their personal, social, and economic relationships in response to the new conditions that influence them; it is increasingly recognized that they must also be helped to do this in ways which harmonize with democratic ideals and conserve democratic values (PEA, 1940, p. 8).

"Adolescence" has historically been a psychological category to be acted upon children by generating ideas about how one lives and should live (Lesko, 2012). For example, G. S. Hall proposed adolescence as a way to respond to mass schooling, industrialization and urbanization occurring at the turn of the twentieth century. The discourses created as a result of previous practices of schooling were no longer working for the new moral order to be created (Popkewitz, 2013). The "adolescents" had to be rescued from their "unlivable" spaces to prevent the disorder. Reforming these kinds became a crucial issue in the Progressive Age, yet never completely "solved". Continuous educational reforms were becoming the commonsense of schooling in the 20^{th} century.

In *Cosmopolitanism and the Age of School Reform*, Popkewitz (2008) argues that educational reforms in the early 20^{th} century directed attention to "the Social Question", which was concerned with the loss of moral order produced by urban conditions. The hope was built into science, and particularly educational science, which was to bring efficiency to society:

> The [reform] programs were to eliminate the social evils of the city by active intervention in the life and conditions of the city. The poor and immigrants were to develop their skills and talents in the modes of living that would undo what was seen as moral disorder (p. 53).

The reforms in the progressive age had a "civilizing mission" against the fear of darkness and incorporated Christian ethics built into government and civic life. It was the "redemption" of urban populations such as immigrants who were not-yet-civilized. There was a similar redemptive narrative in the mathematics education reforms through "mathematical planning" in "democratic" countries for the "promised land":

> Dictators, emperors, and military leaders can set up radio stations, build roads and factories, carry on war, organize educational systems, and control labor and industry without measuring values and relationships. Democracies must deliberately plan such activities for the general welfare, and must use mathematics widely in drafting the plans. Just as surveying and calculating must be thoroughly done before a large bridge can be built, so also must democracy do much measuring and figuring before it can erect its bridges to the promised land (Nygaard, 1939, p. 851).

This was contradictory. While "mathematics" was essential in the secular modern nation, the moral dimension was re-inserted as entailing a missionary purpose in the name of democracy. That is, a moral category was established by human reason that utilized accuracy and precision as a technology of trust. It was the convincing character of numbers in the absence of God in secular democracies that could order particular ways of living to "bridge" with the "promised land". Then, in liberal democracies, the calculable spaces were ordered by the mathematical standards of living were not only to keep the social order but also to maintain moral and ethical qualities of life in the name of “welfare”.

> Society dare not neglect the problem, because its institutions are forever in danger when the uneducated masses become restive by the annoying gap between the things that they can have, and the things that they want. Mechanizing a nation which is not mathematically literate is a dangerous business (NCTM, 1944, p. 230).

The fear of “uneducated masses” and the hope for peace, social-moral order, and “democracy” made the double gesture of schooling possible. It was the task of school reforms to intervene, to re-form those “uneducated masses” who did not have the “faith” in numbers. The concern was less about teaching and learning mathematics, but more related to maintaining social order through ordering proper modes of life that entailed particular moral and ethical values. The fear was not even the prevention of a possible chaos in the midst of the war, but to secure the power relations by creating a set of social and cultural distinctions between people.

2.3. Psychologizing the Other: “Habits of Mind”

Investment to the “mathematical” characteristics of citizens who were going to inhabit the social spaces was part of the nation-building project following the war. The Social Question showed, nonetheless, this investment was not only about regulating the economic life of the people but entailed a redemptive theme that operated through a fabrication mechanism to secure power relations. The socio-spatial management of the masses was in fact part of the “moral” obligation to rescue them from their “improper” or “unlivable” lives in the name of “democracy”, “well-being” or “civility”. While maintaining the social and moral order was part of the “reason” of "math for all", it was not enough to legitimize the practices of differentiating and placing pupils into hierarchized tracks. Creating the social and cultural distinctions between people was necessary but not "rational". As I have argued in the previous chapter, "mathematical ability" was one of the discursive categories that differentiated kinds of people though intelligence tests. To rationalize the social and cultural distinctions while maintaining the importance of mathematical standards of living required different technologies and tactics. And, psychology was the great gift to these mechanisms that simultaneously fabricated particular kinds of people and abjected their Others. The "faith in numbers" was translated into a psychological register, which was in the "habit" of using mathematics in their lives to maintain the "democratic" order:

> Students who are in the habit of formulating real problems, and of insisting on genuine solutions, who know how to judge, collect, and interpret data, who are not misled by inaccurate or misleading statistics, and who know how to recognize valid proof, will not [be] so easily misled by propaganda, suppression of evidence, systemic calumny, demagoguery, or mystical symbols (PEA, 1940, p. 68).

The desire was not only to prevent the citizens of the nation to be manipulated by some kind of mis-evidence but also identifying psychological traits such as the "habit" of utilizing mathematics to formulate problems or to insist on solutions. The crucial question in these reports was the following: "Has he fixed the habit of estimating an answer before he does the computation and of verifying the answer afterward" (NCTM, 1945, p. 197)? Psychology, as representative of scientific knowledge, was to understand the problems, identify their causes, and provide solutions by identifying kinds of people in terms of whether they have "fixed habits" of utilizing numbers instead of "faith". According to Rose (1985), "moral order could be constructed, shaped, organized and re-educated through disciplining the body, imposing habits and regulating through tactics of calculation" (p. 26). Then, identification of individual differences and acting upon them were rather about reformulation of moral treatment.

These "habits of mind" were not only about differentiating and hierarchizing the children, but also a tactic to rationalize tracking and a process of making the lower tracks, which only focused on the application of mathematical skills in daily life, desirable. This was exactly what "math for all" stood for, an "inclusive exclusion" (Agamben, 1998). While this statement embodied an inclusive language such as providing access for all since they were equal citizens, the liberation of individuals was sought in the power relations that differentiated them. That is, while "math for all" was an event securing the power relations, the processes of fabrication and abjection were not embodied in a dialectic account polarizing the Self and the Other. Rather, it was an amalgamation of the differences together with the scientific knowledge that normalizes particular ways of life. It was a mechanism that simultaneously configured the life and policed it.

These mechanisms, as part of the making of the secular modern nation, conjoined with the "mathematical needs" of citizens, taking the life into the target as an object of governance by utilizing the natural finitude of humans, and became a technology of normalizing particular kinds of people who could "adjust" to mathematical modes of life. The "habit of using arithmetic as a normal way of adjusting to life situations" was not only an issue of an individual but also about changing the family to constitute the fabric of a "proper" and "civilized" life in the social milieu. While at the surface this might seem an answer to the question of why learning mathematics by all, the continuous integration mathematical needs of life was not only to govern the life of individual but also the family and community.

> The mathematical needs of the home? diet, economical purchasing, budgeting, social security, transportation, etc.? These did not call for simultaneous equations, the proving of geometric theorems, or for trigonometry. Rather, they called for reasoning and accuracy in the use of arithmetic, of intuitive geometry, and of formulae of the simple type (Douglas, 1942, p. 213).

The "inclusion" of the mathematical needs of home was to rationalize the "math for all". Nonetheless, it was clear that the desire was not to teach mathematics as a subject matter but a way of reasoning to cultivate particular kinds of habits to pursue *the life*. The "educational" subject matter, as reported in the Post-War Plans (1944), was the "body of material that has intimate and demonstrable relationship to the business of living" (p. 227). That is, as Douglas mentioned above, it was not about geometric theorems or trigonometry but a way of reasoning about self and society to establish the accuracy in communications.

In order to ensure the enduring challenge of mathematical life for all to maintain the social and moral order, there had to be different kinds of evidence "such as those of the interview, observation, the examination of work products, and the like" (p. 203). The rich and thick descriptions provided by these "evidences" were supplementary but also as convincing as the numbers. Psychologizing the Other was not as simple as the score gained from an intelligence test. There had to be a consideration of other factors including "sensitivity of using numbers" to establish a category of "normal people".

> Evaluation, like teaching, starts with a consideration of the outcomes, *all* the outcomes, which are to be achieved. In arithmetic these outcomes include more than skill in abstract computation and in problem solving. They include mathematical understandings, mathematical

judgments, the ability to estimate and approximate, habits of use, and the like (p. 203, italics original).

This category of normal people was particularly important from a social point of view where it provided hope of cultivating personalities capable of dealing with everyday situations; who can change themselves in "desirable" directions. The processes of psychologizing, in fact, were to establish a rationale to act upon Others who were "different" than the "normal". These practices were to plan for intervention programs and reform practices that aimed for changing from one state of being to another. Authorization of the category of normal was a product of the amalgamation of distinctions securing the power relations. Considering these desired habits and sensitivities to be cultivated in the making of mathematically able bodies, an "ethico-behavioral schema" was generated to enslave the irrational and sensory aspect of human nature (Wynter, 1995, p. 17). While the formation of the psychological category of normal enabled the action on people in "scientific" ways, it was a reformulation of the moral (or otherwise) conduct of people in the flow of everyday life by ordering proper kinds of habits and sensitivities.

2.4. Planning for Progress and Development: Efficient Curriculum

Pre-post WWII years had encountered a set of complicated practices in the mathematics education field. While the field was in crises in the beginning of 1930s, the mathematics education of all became intelligible towards the end of the war. "Possessing mathematics" had come to be seen as a "powerful asset for responsible citizenship in a world planning for peace" (Fawcett, 1947, p. 199). This had to do with maintaining social stability by regulating the conduct of the masses in the name of "democracy". Although this was a historical reinscription of the "civilizing mission" of numerical operations, the modes of governing assembled with the social and scientific practices of the time and generated the psychological "truths" about children. The depiction psychology as a cultural construction of distinctions formed a specific network of interconnected categories enables a certain kind of understanding of human subjects (Danziger, 1997, p. 181).

Mathematics classrooms became the "great laboratories of democracy" where potential citizens were educated to actively participate in the "democratic ideal" by cultivating the "characters of young people" (Fawcett, 1947). In short, the "democratic way of life" could be organized by curricular planning that was not only "faithful" to make responsible citizens but also planning the future life of "slow learners" or "backward pupils" as citizens:

> The future life of a backward pupil is destined to be quite circumscribed intellectually, and even a limited background of applications helps make him a better citizen. Such an aim [efficient adjustment to life] seems practicable if proper material of instruction is chosen and if there is good teaching (NCTM, 1940, p. 142).

Curriculum planning had started with presumed differences between "slow" and "bright" pupil. The efficient curriculum was the kind of planning that was needed for the masses in order to prepare them to intelligently and efficiently adjust to the fabric of the modern life. The planned curriculum was to cultivate inner qualities of individuals who were to embody particular way of reasoning about the world and the self while investing in the characteristics of humans, as self-governed citizens, utilizing calculations as "practical knowledge" to "be able to" maintain their lives.

> We must provide a more realistic curriculum for the large number of persons who will continue to be absorbed fairly early in life by industry, trade, farm, and business. Then,

too, we must provide a course that will give them greater mathematical security in practical affairs, such as budgets, insurance, taxation, and the like (NCTM, 1945, p. 210).

The will to "possess mathematics" by all children was not only about the "mathematical security" to maintain their lives but also ensuring the "security" of the nation. Then, the discourse of "math for all" was to equip the citizenry with the kind of mathematical competence, which was required for an effective and intelligent living (NCTM, 1945, p. 199). Here, the social mathematics track was perfectly suitable those "slow", "backward" pupils, who were in fact racialized by the colonial reason circulating in the discursive assemblage of school mathematics, to make them an "average child" in the planning of an average American life following the war.

> On the basis of completely trustworthy evidence the claim is warranted that, under competent instruction, American children can and do acquire a satisfactory foundation in arithmetic in the elementary grades, and that the average child, when properly taught, enjoys arithmetic (NCTM, 1945, p. 204)

Once it entered the curricular debates as a patriotic duty in the midst of the war, school mathematics became a fundamental device in planning the life of the masses. Planning the curriculum was to ensure the social and economic uses of mathematics in an average American life: "In curricular planning there must also be considered the quantitative interpretations needed by the adult as a consumer, as a citizen in a democracy, and as a cultural being" (Burr, 1947, p. 58). The discursive assemblage of school mathematics became part of the *administrative machine* of schooling in these years to produce a desirable "cultural being". Then, the task of this machine was to rescue others who deviated from these spaces, which, in fact, reinscribed Enlightenment reason and rationality. The image of the "man of culture", as described previously, became the redemptive narrative for those located in “uncivilized”, “unhappy” and “immoral” spaces. This redemption, at the same time, was possible through working the inner characteristics of children, changing their souls where teachers and curriculum planners “must know how to engender in their pupils sensitivity to the usefulness of number and of measurement in life” (NCTM, 1945, p. 217). It was a reworking of the “sensitivities” of the children to control their conduct through some kind of pedagogical inscriptions as organizing tools and devices of the pedagogical space.

The problem of controlling conduct cannot be ignored. Though the backward child may desire to attract attention to himself, his means of doing so are somewhat restricted. Since he cannot arouse attention through really superior achievement – as superior child can do- he may resort to some crude form of exhibition. In order to be full master of the situation, the teacher needs not only firmness but tact, and should seek to make the simple tasks that are set for the pupils as satisfying and as enticing as possible. The occasional problems of discipline which arise with both superior and dull children should be seen to call for direction and guidance rather than domination, the aim being to develop intelligent self-discipline in an atmosphere of mutual respect for character and worthy achievement. Some of the elements necessary for teachers of backward and superior pupils seem to be natural inherent traits; but study and training are very important in developing these native abilities (NCTM, 1940, p. 137).

The arrangement of the pedagogical space with particular solution strategies such as “using simpler tasks” or “direction” and “guidance” was understood as the “problem” of self-discipline and an issue of classroom management and represented as a continuation of a calm atmosphere of the classroom. Nonetheless, these strategies were not simply a classroom issue or an instructional challenge of mathematics achievement of “superior” or “dull” children. It was about the development of "native abilities" that would fabricate particular kinds of mathematically able bodies who were going to constitute the American race, which was "a form of the imagined unity of the "nation-ness", as Popkewitz (2008) argued. "Math for all" was not only practiced as *an administrative machine* that simultaneously identified and governed two human kinds, but also as *a mechanism of intervention* to restrain those abjected beings lying outside the boundaries of normalcy of this discursively constituted cultural spaces.

3. Doing Mathematics in the Life

The taking of mathematics as "a human creation" (NCTM, 2000, p. 15), rather than as something inherently embedded in nature to be discovered and then possessed for everyday life, is to "allow students to see mathematics has powerful uses in modeling and predicting real-world phenomena" (p. 16). While mathematical modeling practices are to optimize the life opportunities of people of the nation through corporeal regulations, they are, at the same time, part of a larger political distress which legitimized in a developmental logic allowing to act upon children who were identified as either "at risk" or "not-yet-developed" (see Chapter 4). In the contemporary practices, the hope is the preparation of the workforce for the future needs such as information-age technology, more varied, electronic, verbal and mental nature of the work (NRC, 1989). While, at the surface, the transformations in the political economy (i.e. from industrial work to the information technology) might seem a reasonable reason of "math-for-all", these also have to do with the demographic changes in the tracking system where the number of students is increasing in the lower tracks and the numbers are decreasing in the higher tracks. These changes are also complemented with the fear of "a divided nation" as the racial distribution across the tracks becomes disproportionate (p. 14). The discursive practice of "possessing mathematics for life" has started be unfeasible in the contemporary society. Mathematizing the real world phenomena and doing mathematics in real world situations become the new salvation narrative of school mathematics. This reformulation requires new technologies and tactics to make "math-for-all" intelligible by re-identifying the "mathematical needs" and re-configuring the mathematical life for the last few decades: "Ambitious standards are required to *achieve a society* that has the capability to think and reason mathematically and a useful base of mathematical knowledge and skills" (NCTM, 2000, p. 29, my italics).

3.1. Mathematical Needs of the Changing World Order

3.1.1. Socio-spatial Configuration of Mathematical Life, 1980s-present

In describing "the need for mathematics in a changing world", the emphasis is concentrated on the widespread availability of quantitative information in everyday life, which involves "making purchasing decisions, choosing insurance or health plans, and voting knowledgeably all call for quantitative sophistication" (NCTM, 2000, p. 4). While needs are again at the heart of the everyday life of human beings, we start to see the practice of decision making as a highlight in these discursive statements (Heyck, 2015). That is, the mathematical needs are not merely a civilizing medium between the self and the life as separate systems but constitute an inseparable part of the system where the body has to referentially fit. This system has to produce rational decisions to shape and fashion the corporeal acts in the flow of daily life where children are to "be able to give a rationale for their decision" (NCTM, 2000, p. 36).

So, what makes these changes possible? I have already argued in the previous chapters how mathematical modeling practices have emerged in the assemblage with the changes in the modern social thought from decider to decision in the process where rational choice seems more hopeful than human reason itself. In the "reason" of "math-for-all", not differently, we can trace an emerging *model* of man whose image is not a static picture but a fluid visual, representing an inventive and adaptive problem solver yet bounded with rationality, what Heyck (2015) terms as "homo adaptivus" (p. 82). The mathematical needs of the 21st century are no longer the ability to add or subtract since the calculators are doing that kind of practices. Desired for an "effective problem solvers" is their ability to "constantly monitor and adjust what they are doing" (NCTM, 2000, p. 54). Conceptual understanding becomes essential for all, not only the selected few, since what was previously instructed in the lower tracks are done by the machines in today's world. The task is not teaching how to add and subtract but how to be flexible in reasoning in new and to some extent uncertain conditions:

> The requirements for the workplace and for civic participation in the contemporary world include flexibility in reasoning about and using quantitative information. Conceptual understanding is an essential component of the knowledge needed to deal with novel problems and settings. Moreover, as judgments change about the facts or procedures that are essential in an increasingly technological world, conceptual understanding becomes even more important (NCTM, 2000, p. 20).

In the pre-post WWII period, life was regarded as quantitative, too. Nonetheless, the aim was to prepare students for already determined quantitative contexts such as taxation or insurance, which had to be done either with counting or measuring. Today, although these economical grounds are still under consideration, the conceptual understanding of mathematics is necessary in order to "deal with the novel problems and settings". Children are to show their ability to mathematize any real world phenomena. This, however, requires a modification of the cultural thesis for mathematically able bodies moving from a strict reasonable self towards more flexible, adaptive and fluid selves.

At this point, in order to make sense these changes regarding the quantifiability of the non-economic (or any) spheres of life, we can situate the increasing demand for conceptual understanding within the conditions of political economy since the economic crises occurred in the late 1970s. As an important change in the practices of the market relations, the neoliberal rationality is disseminated and it is becoming part of all domains and activities where money is not even the issue (Brown, 2015). When the practices of the liberal reason begin to create problems for the contemporary world, new technologies and tactics have to be invented to reconfigure the mode of governance to maintain stability and social order. "An agenda for action" had to be devised (NCTM, 1980). These actions, however, were about the embodiment of particular corporeal regulations in the flow of everyday life and the preparation of children as modern citizens through the re-configuration of mathematical needs. For example, statistics and data analysis have entered into the curriculum standards as one of the content domains to secure the uncertain futures by governing the present:

> Statistics, the science of data, has blossomed from roots in agriculture and genetics into a rich mathematical science that provides essential tools both for analyses of uncertainty and for forecasts of future events. From clinical research to market surveys, from enhancement of digital photographs to stock market models, statistical methods permeate policy analysis in every area of human affairs (NRC, 1989, p. 5).

The data tools to analyze uncertainty have become the new needs to be invested in to maximize the capacities of the human capital located in the biopolitical field forming ability-machines that are responsible for their own selves. The developmental machinery has also entangled with these processes where the continual and never-ending investment by particular mathematical capabilities becomes the desired practice. These practices have to do with the alterations of economic activity. Nonetheless, this is neither a complete flight from the previous mechanisms nor a radical departure from the mode of production. They historically entangle with the making of desired humans. Then, the changes are about extending the field of power relations to make a particular kind of self in making a program for the rationalization of society and an economy. That is, the analysis has to do with studying the way human capital is formed and accumulated; as a result, applying this formation of economic subject into new fields and domains spreads across the meticulous details of life.

American neo-liberalism has revealed a more complete and exhaustive appearance, according to Foucault (2004), including the generalization of mode of reasoning throughout the whole social milieu that bodies are part of. That is why, to reconfigure these ontologies, a reformulation of the "mathematical" needs is necessary as this field contains rather uncertain futures and the boundaries cannot be known in advance. Modes of mathematical thinking and reasoning have to be embodied by the mathematically able bodies in order to "mentally adapt" to the new socio-spatial configuration of the life.

> Communication has created a world economy in which working smarter is more important than merely working harder. Jobs that contribute to this world economy require workers who are mentally fit—workers who are prepared to absorb new ideas, to adapt to change, to cope with ambiguity, to perceive patterns, and to solve unconventional problems. It is these needs, not just the need for calculation (which is now done mostly by machines) that make

> mathematics a prerequisite to so many jobs. More than ever before, Americans need to think for a living; more than ever before, they need to think mathematically (NRC, 1989, p. 1).

While the text is more concerned about the jobs in the contemporary world, we can easily trace how the desire for "mentally fit-workers" who can think mathematically spreads in the meticulous details of life of the human beings "more than ever before". These details are not named, what is configured is not life but the model of the possible life. That is exactly the point of these discursive statements: The ongoing and undefined mathematical thinking in the face of uncertainty requires continuous self-investment in mathematically able bodies who can anticipate, act flexibly and change themselves easily to the unforeseen circumstances. This is nonsensically paradoxical yet governs the present by inserting the future as a category that can be acted upon. The continuous self-investment, in fact, domesticates human conduct across present and future: "Learning with understanding is essential to *enable students to use* what they learn to solve the new kinds of problems they will inevitably face in the future" (p. 21, my italics). The contemporary mathematical needs, then, include "being able to reason mathematically" to become a "powerful citizen" for the new century:

> The mathematics that people need is not the sort of math learned in most classrooms. People do not need to regurgitate hundreds of standard methods. They need to reason and problem solve, flexibly applying methods in new situations. Mathematics is now so critical to American citizens [...] If young people are to become powerful citizens with full control over their lives, then they need to be able to reason mathematically-to think logically, compare numbers, analyze evidence, and reason with numbers (Boaler, 2015, p. 7).

This transmutation toward reasoning with numbers has to do with the reconfiguration of mathematical needs to make people as powerful citizens and the modification of the practices of political economy with neoliberal rationalities and marketization processes. While some might argue that contemporary mathematics education reforms are aligned with the new right, neoliberal, and neoconservative racial projects (i.e. Martin, 2013), this is not a direct application of liberal or neo-liberal market logic onto the discursive assemblage of school mathematics. On the other hand, it is more about the exercise of power relations, which undeniably has an economic horizon of enactment, in a more diffuse and active field that produces normative relations and hierarchical categories in every detail of life accompanied with the historical problematic of human needs.

3.1.2. The Collective Will to Tap the "Power" of Mathematics

"To participate fully in the world of the future, America must tap the power of mathematics" (NRC, 1989, p. 1). These very first words of the *Everybody Counts* aim to design the future state of mathematics education practices, which re-enunciate a collective hope for social security and progress not only within the country but also across the world.

> In today's world, the security and wealth of nations depend on their human resources. So does the prosperity of individuals and businesses. As competitors get smarter, our problems get harder. Long-term investment in science and technology—both for businesses and for our nation—requires serious commitment to revitalizing mathematics education. It is time to act, to ensure that all Americans benefit from the power of mathematics (NRC, 1989, p. 2).

In *Age of Fracture*, the historian Daniel Rodgers (2011) argues that we are at an age "with a sharp and insistent sense of power" as partially constituted by the movements of 1960s, which have produced sets of new languages and practices across the political landscape. "Power moves were everywhere in the economic analysis of politics", he writes, "but what propelled those moves were not the needs of the bloc interest groups, as the pluralists had imagined it, or interest of a new knowledge class, but the microphysics of individual political action" (p. 85). The Algebra Project, for instance, as an exemplar of grassroots empowerment project in mathematics education, has emerged as mathematics literacy program for Black communities to ensure economic and civic equality of all.

> In today's world, economic access and full citizenship depend crucially on math and science literacy…absence of math literacy in urban and rural communities throughout this country is an issue as urgent as the lack of registered Black voters in Mississippi had in the 1961 (Moses & Cobb, 2001, p. 5).

Part of this project aimed at challenging the unfair tracking practices in public schools and teaching mathematics in equitable ways, yet there was another demand of the movement in which "Black people challenge themselves", too (p. 81). Mathematics was a tool of liberation. Mathematical literacy becomes a *key* to freedom and a tool for empowerment. In this project, the tackling questions were: "Who can be a citizen? What are the requirements?" (p. 68). The answers, nonetheless, have to do with the historical narrative of making the American race. It is claimed that mathematics literacy was a key requirement to be a citizen of US for Black people. Therefore, the project functioned beyond the intended political action, which was to stand against tracking and segregation practices. It also had a civilizing mission where mathematical power was inserted as a regulative strategy to make up Black people as citizens of the United States, to shape and fashion who they are and who they should be.

While this civilizing manner is visible as a historical reiteration of "math-for-all" in making the self and the society, these practices are also generative in challenging the public intellectual life since the WWII. The conceptions of human nature are changing; now the emphasis is on choice, agency, performance and desire (Rodgers, 2011). That is, we have started to see more about individuals, contingency, and choice although we cannot exactly trace any identity in a single category. This story, according to Rodgers, was about "how Americans reimagine themselves and their society" in the conjuncture of:

> … the economic crises of the 1970s, the new shape of finance capitalism and global markets, the struggle to hold identities stable where race and gender proved unnervingly divisive, the linguistic turn in culture in an age of commercial and malleable signifiers, the nature of freedom and obligation in a multicultural and increasingly unequal society, and the collapse of Communism (p. 10).

This re-imagination becomes possible through embodying an anticipatory reason; not identifying the "slow pupil" but "children at risk", not picturing "the man of culture" but envisioning "adaptive ability-machines", not with the single mathematical truth but multiple mathematical truths and/or mathematical experiences. In the midst of these uncertainties to imagine a future together, the constructivist project has emerged as both a scientific project and a reform movement to question the Cartesian assumptions of the mind. Nevertheless, the departure reconfigures the liberal autonomous subject who continuously invests for his living capacity of mind in which the boundaries of growth cannot be set in advance by mathematical structures. In fact, these attempts are to re-inscribe the Cosmopolitan notions of self such as liberal autonomous human subject but in unfinished forms (Popkewitz, 2008).

In the constructivist movement, nevertheless, the mathematical experiences of students in their everyday life have emerged as partially constitutive in the building of new knowledge. As NCTM (2000) points out in explaining the learning principle: "Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge" (p. 20). It is "not just the absorption of others' knowledge" (Romberg, 1992, p. 433). While this has generated a discussion regarding "whose" knowledge and experience is represented in the curriculum and instruction (Apple, 1992), the concern of "prior knowledge and experiences" in research and reform practices has also been prolific in producing new tools and technologies not only to distinguish the two human kinds in "math-for-all", but also to generate a capacity to act on them. If their experiences or prior knowledge are not-yet-developed for whatever reason, students are at risk and they have to be supported to be part of the collective.

> Students' repertoire of tools and ways of communicating, as well as the mathematical reasoning that supports their communication, should become increasingly sophisticated. Support for students is vital. Students whose primary language is not English may need some additional support in order to benefit from communication-rich mathematics classes, but they can participate fully if classroom activities are appropriately structured (NCTM, 2000, p. 60).

To be able to communicate mathematically, "students whose primary language is not English" have to be supported to participate and to benefit from the "richness" of instruction. This is just one example. Thanks to statistical technologies that emerged in the late 19th and early 20th century, the categorical identification of the child has become pervasive even though public intellectual life has begun to be shaped by more intersectional and elective subjectivities where the emphasis has been on agency and performance. While this historical entanglement reveals the (im)possibility of freeing agency and eventually essentializes subjectivities in terms of what they do rather than who they are, an individualism occurs not at the level of a single identity but differentiating the social space. Then, there remains the will to mathematically empower those whose sensibilities are partitioned in order to maintain the "democratic" order (Rancière, 1999). The "additional" support for those children, who are "at-risk", is not only to invest the individual human capital to make autonomous liberal human subject, but also to participate the classroom activities and simultaneously to become part of the democratic society. That is, the tapping the "power" of mathematics is not only about making the child as a single mathematically able body, but also as a technology of the self to collectively participate in the formation of a society, that contains mathematically able bodies. It is part of the building a sense of mutual obligation to hold a national community together (Rodgers, 2011).

3.2. Democracy in and through "Math-for-All", 1980s-present

In the contemporary discourse of "math-for-all", one of the important points is to provide additional support for those students who need it in order to make the mathematics education field more democratic and egalitarian. As reported by NCTM (2000), "a society in which only a few have the mathematical knowledge needed to fill crucial economic, political, and scientific roles is not consistent with the values of a just democratic system or its economic needs" (p. 5). Although there was a similar argument in the reforms during 1940s regarding the inclusion of those who had been excluded before to establish a "common ground" for an efficient and intelligent citizenship in hierarchized tracks, the practices of the present differ since the ability-grouping is no longer recommended. The new practices are more concentrated on ensuring the participation of all by providing a democratic access to powerful mathematical ideas. This political distress has to with not only "the preparation of a populace for participatory citizenship" but also about "a moral commitment to the common good" (Malloy, 2002, pp. 17-18).

3.2.1. Democratic Subjects of the Bright Mathematical Futures

One of the reasons that make the concern of democratic access to powerful mathematical ideas possible is the "great" political distress, fear and concern about the "risk of becoming a divided nation". The problems that are produced by the previous practices of mathematics education and coupled with the historical concerns regarding the stabilized notions of "mathematics" and "mathematical ability," are reflected as a threat to American "democracy". Nonetheless, what have been circulated in the mathematics education field are the immobile inequalities as ordering practice to correct "mathematical illiteracy":

> We are at risk of becoming a divided nation in which knowledge of mathematics supports a productive, technologically powerful elite while a dependent, semiliterate majority, disproportionately Hispanic and Black, find economic and political power beyond reach. Unless corrected, innumeracy and illiteracy will drive America apart (NRC, 1989 p. 14).

This is not a merely an economic concern or territorial issue of the state, but a political anxiety. "Mathematical illiteracy" or "innumeracy" of particular groups is envisaged as a threat revealing "alarming signals for the survival of democracy in America" (p. 14). Individual difference, as a fact, is inserted in the social milieu of mathematics education as not a problem of equality and justice but a technology of identifying those who do not belong to the whole. The fixed inequalities as determined by the mathematical literacy tests or other mediums are represented as the initial problem and starting point to "empower" those less than (Rancière, 2010). The solution of the so-called "achievement gap" is not easy albeit the method is known and can be planned: Mathematical empowerment of those identified as illiterate or innumerate as citizens of a democratic society. The "power" of mathematics has to be (re)distributed to those who are "lack of", "mathematically illiterate" or "innumerate" in order to ensure the equality and homogeneity. This is what Rancière (1999) calls as "policing", which is not so much about the disciplining of bodies as a rule of governing but "a configuration of *occupations* and the properties of the spaces where these occupations are distributed" (p. 29, italics original). Let me expand this a little bit. There is the desire to tap the "power" of mathematics to make citizens as liberal autonomous subject who is prepared for the future uncertainties where children are to continually invest their own capital to form the collective whole. Children are not asked to absorb the predefined set of mathematical skills, but to embody mathematical reasoning to develop the ability to calculate the unforeseen circumstances in the meticulous details of their lives. For example, as reported in *Adding it Up*, "citizens who cannot reason mathematically are cut off from whole realms of human endeavor. Innumeracy deprives them not only of opportunity but also of competence in everyday tasks" (p. 16). Questions in relation to the regulation of life can be pursued: What does being cut off from whole realms of human endeavor mean? How does numeracy inserted in everyday tasks regulate life? What is the function of "innumeracy" in forming the collective whole? Then, "innumeracy" is not about teaching or learning mathematics but functions as a tactic to distinguish two kinds in this collective whole and a technology to demarcate the ways of being, acting or participating without exactly saying what to do but specifying how to do. That is, "mathematical reasoning" is moving along every sphere of the life as a normative mechanism to control the daily acts without telling what to do. Mathematically able bodies is the *police,* not as a solid figure keeping the gates, but an illusory network of practices that generates material consequences and microphysics of power relations such as to govern the conduct, make particular subjectivities and produce mechanisms of abjections.

How all these are related to "democracy"? If one accepts the logic that the police are responsible to work for man and his well-being, security and happiness, which is also consistent with the "pursuit of happiness" of the American narrative, he has to presume that "there are patterns and procedures of ruling that are predicated on a given distribution of qualifications, places and competencies" (Rancière, 2010, p. 53). In fact, the importance given to "mathematics" does exactly come from this kind of organization. For instance, Jo Boaler (2015) argues that "we need to bring *mathematics* back into math classrooms and children's lives, and we must treat this as a matter of urgency to improve our children's and our country's futures" (p. 10, italics original). What does it mean to bring mathematics back not only to the mathematics classrooms but also children's lives? Aren't the underpinnings of the life already mathematical as NCTM (2000, p. 4) reports? Then, we have to recognize that what is at the issue is not only a preparation of an already configured mathematical life, but also an anticipation of uncertain futures where the lines between life and mathematics cannot be drawn. The police of mathematics education become the mathematically able bodies as body of knowledge, which have the capacity or have the permission to permeate every detail of public and private life. That is to say, the line between public and private life blurred in a sense of privatization of the public life. Now, deciding between insurance and health plans has become an issue of individual happiness, rather than a public concern. These future-oriented reforms are exactly for the institutionalization of democracy through children's *bodies* for their happiness and pleasure, not in a top-down way but by evoking interest, need and pleasure to be able to reason mathematically:

> The advent of new technologies means that all adults now need to be able to reason mathematically in order to work and live today's society. What's more, mathematics could be a source of a great interest and pleasure for Americans [...] to prepare them for the future (Boaler, 2015, pp. 4-6).

Then, mathematically able bodies are neither passive recipients nor are they subjugated by the police. On the contrary, the deployment of mathematics in various life contexts, which does not have to do with schools only, might suggest how the discursive assemblage of school mathematics proliferates, innovates, annexes, creates and penetrates bodies in an increasingly detailed way and controls the populations in an increasingly comprehensive way (Foucault, 1978, p. 107). So, making democratic citizens embodying mathematical reasoning does not so much have to with the concern for an egalitarian society, but it is to occupy the public life with private interests.

If democracy is a form of relationship that defines a specific subject, according to Rancière (2010), it is far from a political commitment, but a form of government with a definite set of rules and institutions to reduce the political action. This reduction, in fact, leads to empowerment of "private life" or "pursuit of happiness". This is what exactly has been targeted for the contemporary practices to maintain the social order and to secure the power relations. The more privatization of the public sphere yields to the less political action since the subjects have an "interest" or "pleasure" to maintain this privatization. The transformation of the public sphere into a private one, not only in monetary terms but also as an issue of self-interest, might not have anything to do with the representation of the voices or the delimitation of the particular groups' orders over others' lives. It is, however, to make the democratic subjects as the police of their bare lives. That is why, mathematically able bodies, as body of knowledge, are not only police of the discursive assemblage of school mathematics but also product of it as kinds of people to regulate their own lives as subjects. Giorgio Agamben (1998) explicates these blurred maps of life in relation to the making kinds of people as both objects and subjects of the democratic society where both forms simultaneously move towards "the new biopolitical body of humanity" (p. 9).

3.2.2. Establishing Mathematical Consensus and Building Mathematical Futures Together

In contemporary reforms, the desire for a society that could be united even its diversity is historically retold. The enunciation of the will for democracy, nonetheless, is no more about ensuring a common (mathematical) ground across all citizens but more concentrated on building *consensus* with different actors to ensure the collective existence of people even despite their "differences". New tools and technologies are to be invented to maintain the social order and decent living in the society.

> The challenge we now face is how to create a curriculum filled with responsible social and political issues that will help students understand the complexity of such problems [of nuclear waste or deforestation], help them develop and understand the role of mathematics in their resolution, and allow them, at the same time, to develop mathematical power (Romberg, 1992, p. 435).

The will to tap the "power" of mathematics, then, is important not only for individual happiness but also it becomes a larger concern where the emphasis is on the incorporation of "real social situations" into mathematics curriculum and instruction in order to enable the livable life. These practices legitimize the *anticipatory actions* of and for people, which include adequate development or support for those who are at-risk in the making of the child as responsible democratic citizen. The rationalization of embodiment of mathematical reasoning to do mathematics in the unforeseen circumstances is to anticipate a future uncertainty in the liberal democracies. This uncertainty is not something merely a threat against the administrative machines, but something promising about life itself, which has been the site of action to secure the power relations in the (neo)liberal societies as well as to provide "uncertain" spaces for self-fashioning individuals (Anderson, 2010).

While the threat and promise are deployed in liberal societies to enable the anticipatory actions for and of people, this network of relations reveal the "democratic paradox" in Rancière's term. That is, although the task is to ensure the liberty and equality of people, there is the governmental mechanism that institutes rules and norms for social participation. It has to find a "common" language for "common" people as it does through the embodiment of hopes and fears circulated across the discursive assemblage of school mathematics. What I have written about the participation in model eliciting activities toward the end of Chapter 3 is equally valid here. To restate briefly, the contemporary pedagogical models have to invent new languages and tools, such as sociomathematical norms that are established by children themselves, to enable "diverse" groups of people communicate with one another. Nobody needs to have a mathematical common ground as a precondition to participate in these communicative discourses; nonetheless, they have to participate effectively to reach an agreement produced by their own groups. As I have argued, this has less to do with the mathematics as a subject matter but includes the ability to make persuasive arguments and justifications to convince others. It is the moral economy of the mathematics classrooms where students are obliged to decide the "truthiness" of the mathematical narrative and reach a consensus with the most persuasive argument (Daston, 1995). Additionally, this ability has historically been put into a developmental continuum to categorize the differences in a hierarchical scale. Then, the institutionalization of the sociomathematical norms of participation reveals a fundamental inconsistency: A paradoxical democratic mechanism, which simultaneously promotes individual autonomy while it homogenizes the space by making the discourse smoother to be able to reach a consensus. Put differently, the effort to socialize students in the name of democracy is to foster the private interest of the child in the ongoing classroom discourse. This is problematic in at least two points. First, although the learning community is constructed based on a taken-as-shared knowledge, what counted as an acceptable mathematical explanation is regulated by the sociomathematical norms that justify the truthiness of the arguments and that show the degree of persuasiveness (Yackel & Cobb, 1992). If this space is regulated by these norms, which also is put in a hierarchical continuum from personal towards mathematical basis, one cannot argue that the community is egalitarian at all. If a consensus is reached in this (learning) community, it has to occur in these normal and abnormal spaces, creating a doublet of subjectivities, which are constructed through a "scientific" basis and an inclusive language, yet revealing an unethical socio-spatial partitioning. This relates to the second problematic point, which is about the "democratic individualism" that Rancière (2006) argues: "It is a certain collectivity, the well-hierarchized collectivity of bodies, milieus, and 'atmospheres' that adapt knowledges to ranks under the wise direction of an elite" (p. 305). This elite might not necessarily be a wealthy person, a moral ruler or a sovereign; on the contrary, it is the landscape of power/knowledge relations that have a program of remaking oligarchic ruling with "democratic" tactics and tools (i.e. self-interest in the making of public) in (neo)liberal societies to specify a subject who is able to live in this regime.

Cultural theses for the citizens of the reforms in mathematics education reveal a human kind who is not fixed but flexible. A human kind who is to be able to adapt into the various contexts as autonomous, collaborative individual, problem solver and decision maker is anticipated and modeled. Coupled with the undesirability of tracking students in isolated spaces, communication in mathematics classrooms gains an importance not simply because teaching and learning the subject matter, but it is more related to organization of the spatial culture for the all, which necessarily includes the self, the other and their relations with one another.

In the creation of environments that fosters discussion and collaboration, the subject of "mathematics" becomes an illusion and transmogrifies into a tool to make an administrable citizen (Popkewitz, 2004). In order to create a homogenous space of taken as shared meanings and to reach a consensus, the beliefs and values of individuals need to be cultivated in particular ways not only to maximize their "learning" opportunities but also to contribute the common good. The Social Question is re-posed to save the children from their unlivable lives that might disturb the "democratic" stability yet it simultaneously evokes the historical tension of losing the moral order in the city. The police are lucky to have new psychological tools and technologies to maintain the "mathematical" qualities of life and to restrain the "pathologic" cases that might disrupt the harmony.

3.3. Psychologizing the Other: Mathematical Mindsets

Productive disposition is described as "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" as one of the strands of *mathematical proficiency*, which has not been developed across the American youth in the sense that the writers define it since the last quarter of the century (NRC, 2001, p. 5). Although it is possible to trace the "usefulness" of mathematics since the Enlightenment, there is something to recognize in the contemporary discourse. The mathematical proficiency, which is to be habituated by *all*, is articulated with "adaptive reasoning", "strategic competence", "conceptual understanding" in addition to the "procedural fluency" (pp. 136-138). No longer the possession of specific mathematical skills for the preservation of the determined democratic way of life is required as in the 1940s, but what needed is to create bright mathematical futures with an embodiment of mathematical reasoning in every sphere of life by the adaptive kinds. In order to fulfill these needs, nonetheless, the present has to be governed by the pedagogical models and tools employed to change the child from one state of being to another such as inspiring the children from not only their individual "painful" experiences but also to recover the "mathematical trauma" of the American society.

> Together we can inspire children who in turn will go on to create a brighter future for their children, filled with the scientific, creative, and technological discovery that mathematics enables. Let us move together from the mathematics trauma and dislike that has pervaded our society in recent years to a brighter mathematical future for all, charged with excitement, engagement and learning (Boaler, 2015, pp. 194-195).

Historically, pointing out the psychological constructs (e.g. dislike, excitement) has always been a way to talk about "differences" (Danziger, 1997). Mathematics education field is not an exception. "Habits of mind" of seventy years ago is transmogrified into "mathematical mindsets", which is embodiment of "habitual inclination" for mathematical thinking and reasoning. Disagreeing with the inherent-ness of "math brain" or "math gift" from the birth, opportunities are to be created for the students and teachers to allow them to "develop strong mathematical mindsets" (Boaler, 2016). While this disagreement is an important way of thinking about the unfair practices, it can be easily trapped into the "reason" of "math for all": There exists a pair of students and one of them needs additional support to be provided.

It is not enough for a student to be successful in mathematics but he or she has to "recognize the value of studying mathematics" and "believe that they are capable of learning mathematics". In brief, the positive attitudes, beliefs and values are to be developed as part of creating mathematical mindsets. While this reveals the contemporary configuration of what it does mean to be (mathematically) successful, which does not include the achievement only, it is, at the same time, a reformulation of Cosmopolitan self as an unfinished, lifelong learner (Popkewitz, 2008).

> This conviction increases students' motivation and willingness to persevere in solving challenging problems in the short term and continuing their study of mathematics in the long term. Interest and curiosity evoked throughout the study of mathematics can spark *a lifetime of positive attitudes toward the subject* (NCTM, 2014, p. 8, my italics).

The trigger for "a lifetime of positive attitudes toward mathematics" is not due the radicalism of mathematical mindsets as opposed to traditional approaches, as Boaler (2016) argues, but more related to shifting practices in the discursive assemblage of school mathematics to cultivate a model of man whose image is not a static picture but a fluid visual, representing an inventive and adaptive modeler. The spatiotemporal configuration of mathematically able bodies in the contemporary practices suggests an open, never-ending and continuous mathematics user as ability-machines, which has an affective dimension to disperse across the capillaries of life.

> Successful math users have an approach too math, as well as mathematical understanding, that sets them apart from less successful users. They approach math with the desire to understand it and to think about it, and with the confidence that they can make sense of it… They approach math with a *mathematical mindset*, knowing that math is a subject of growth and their role is to learn and think about new ideas. We need to instill this *mathematical mindset* in students from their first experience of math (Boaler, 2016, p. 34, italics original).

The principles and rules of knowledge that historically has been constituted have generated once again psychological categories for children as a dividing practice. The desire to understand mathematics is reformulated into mathematical mindset to distinguish the "growth mindset" as opposed to "fixed mindset". The instillation of mathematical mindsets is to cultivate inner characteristics of children to evoke a desire and pleasure by themselves to use mathematics in every sphere of their lives.

This regulatory grid of intelligibility produces normal categories to reconfigure the self as related to Other. The identification of these categories does not necessitate someone outside of the realm of these practices. It is the one who needs to monitor his or her progress and to assess whether he or she is on the "track" as planned. One of the "strategies" to "help" students become aware of their learning processes is the self-assessment practices where they "are given clear statements of the math they are learning, which they use to think about what they have learned and what they still need to work on" (Boaler, 2016, p. 151). The continuous and never-ending assessment of the self is a new invention compared to seventy years ago. No longer is the ability to transfer the possessed mathematical knowledge being evaluated by someone, but rather, the crucial work is to "develop metacognitive awareness" so that students are to think of themselves as learners of mathematics, problem solvers and mathematical thinkers to evaluate the reasonableness of their arguments and their performance either in school or in their own lives (NGA & CCSSO, 2010; NCTM, 2000; 2014). The cultivation of self-rule is desired rather than positing the categories of normal and abnormal for those who "need" to follow. These practices have more to do with the emergence of technologies of security, mechanisms of social control and reconfiguration of populations as both objects and subjects of their behaviours (Foucault, 2004). As Deleuze (1992) points out, in societies of control, while disciplinary systems underwent a crisis, the new forces are gradually instituted and different control mechanisms are built almost equal to the harshest of confinements despite the expressions of new freedom and flexibility (p. 4). Then, the development of lifetime positive attitudes crosses the boundaries of school by securing the never-ending process of learning and by ensuring continuous self-evaluation. Nevertheless, this is less about contestation of categories of normal but more related to different strategizing the power relations in the biopolitical field. Then, the question becomes interested in the child's family life that would produce human capital, the type of stimuli, form of life, and relationship with parents, adults, and others can be crystallized into human capital (Foucault, 2004, p. 230). The strategies to secure power relations reveal the paradoxes of the contemporary practices. While there is an effort to expand the possibilities and to mobilize individuals with uncertain futures, the mechanisms of control are built to restrain those possibilities. Then, what is say-able, think-able and live-able are not open to future imaginations but historically bounded with the reason and rationality that make up people.

3.4. Planning for Progress and Development: Modeling the Efficient Teacher

The discursive analysis of mathematics education practices makes visible the institutionalized discipline and control mechanisms that historically organize mathematically able bodies, both as a body of knowledge and particular kinds of people, to achieve a society that is capable of thinking and reasoning mathematically. There are also cracks in this assemblage such as a paradox of democracy or anticipation of uncertain futures through calculations in the present. Nevertheless, the changes in the mathematical needs of the twenty-first century and practices and re-calculation of the (un)livable spaces historically reiterate the administrative machinery of "math for all" that simultaneously identifies two human kinds in the social milieu in relation to one another while it produces a mechanism of intervention to save and rescue those located in abjected cultural spaces. Then, there needs to devise a plan for the children who are "less" than their peers in their "performances" of doing mathematics. However, the plan, the mechanism of intervention, cannot be in isolation of "different" kinds. Having produced sociological and political problems across the country, which is signified in the fear of "a divided nation", tracking has to be eliminated. At the same time, contemporary mathematics education practices have to invent new tools and technologies not only to prevent this "divide" but also to strengthen the ability to live together given the "diversity" to maintain social and moral order in public-private spaces. While these tools are entangled with the will of "mathematical empowerment" for democracy, there is a need to plan the classroom communicative discourse to generate the productive citizens who are not only mathematically capable but also have the ability to communicate with one another.

The effective planning of participation structures does assemble with the principles and "reason" of schooling where the subject matter is translated in order to plan effective pedagogy with an assumption of the discursively formed psychological truths about children (Popkewtiz, 2004). The effective pedagogy is to *model* a possible teaching to make the effective teacher to "manage" the diversity. That is to say, planning is not eliminated yet transmuted into to the organization of "a diverse array of students' responses" in order to "promote productive disciplinary engagement" for all students in the same classroom space (Stein, et. al., 2008, pp. 314-315).

Contemporary discourse reveals an unquestionable importance to the (mathematics) teachers as professionals who have faith in the sciences of pedagogy. These "newly" emerged practices are important in two points. First, the socio-spatial management of populations is territorialized into the classrooms as modeling "efficient" pedagogy. Second, relatedly, while teachers are regarded as the agent of the effective planning and someone "who is in control", they become the objects in the research and reform practices, fabricated as "engineers" of the learning environment.

The desire to "lead" children toward more powerful, efficient, and accurate mathematical thinking is central in the lesson where teachers are to anticipate the possible student inputs, to prepare responses and to make decisions about how to structure these diverse mathematical contributions so that the mathematical agenda of the lesson is progressed and furthered (Stein, et.

al., 2008). While the anticipation of a wide variety mathematical contributions are partly controlled by the mathematical learning trajectories, reasoned developmentally that put the children's thinking into a (hierarchical) order, the integration of "useful" ideas into a single package is to provide "a model for the effective use of student responses" that is potentially making "such teaching manageable for many more teachers" (Stein, et. al., 2008, p. 314). In this cycle of teaching, as reported in the *Benchmarking for Success* (2008), "the major elements of what can be thought of as the "instructional delivery system—the people, tools, and processes that translate educational expectations into teaching and, ultimately, into learning for students" (p. 23).

In this system, modeling teaching is a practice to manage the diverse thinking of children, to respond if necessary and to provide "additional support" for those who needed and simultaneously populated into sociological categories. As reported by NCTM (2000), the standard for equity requires *accommodating differences* among students "who come from diverse linguistic and cultural backgrounds, who have specific disabilities, or who possess a special talent and interest in mathematics" (pp. 12-14).

Engaging in a management process that includes anticipating, monitoring, selecting, sequencing and connecting the diverse range of students' responses generates an image of teacher as decision maker in the system and facilitator of the process. While the aim of this facilitation process is to nurture "students' mathematical authority" (Stein, et. al., 2008, 332), teacher remains as someone who is in control and "makes judicious choices about which approaches to be sure to select for class discussion" (p. 334). This requires, selecting and ordering mathematical responses in a particular sequence to make the discussion "more mathematically coherent and predictable", which concurrently assembles with the developmental reason, "rather than being at the mercy of when students happen to contribute an idea to a discussion" (p. 328). There has to be a balance between accountability and student agency since too much emphasis on "student authorship" can lead to "classroom discussions that are free-for-all" (Smith & Stein, 2011, p. 2).

Moral dimension is not spoken but historically reiterated. While the hallmark and starting point of effective teaching practices is the focus on students' prior mathematical knowledge and experiences, the teacher is the agent who "actively shapes the ideas that students produce to lead them toward more powerful, efficient, and accurate mathematical thinking" (Stein, et. al., 2008, p. 320). Nevertheless, the historical account of accuracy shows that these are the practices that do not necessarily yield towards merely learning mathematics as a subject matter or doing a creative and rigorous work, also configure kinds of people who are "able" to live as the moral actors and as potential truth-tellers of a particular time-space dimension, which reveals close association with Enlightenment reason and rationality. Then, assisting children who are identified as "at risk" or "not-yet-developed" with an additional support is not only an issue of national progress but also a concern of maintaining the social and moral order.

Distinguishing of two types in the realm of "math-for-all" also assembles with the populational reasoning that enables these different kinds are populated into stabilized categories such as race, ethnicity or gender. While "attending to access and equity" is a worthwhile effort, one has to recognize what is taken for granted. That is, if "attending to access and equity means recognizing that inequitable learning opportunities *can* exist" (p. 60, my italics), if the work of equity is to start with a priori statement that is to be "solved", if the inequality becomes the ordering practice of teaching, reform or intervention, the effort to attend access and equity becomes limited even dangerous since those located in such categories (i.e. female, English language learners, members of other minorities) become the object of research to be "saved" or "rescued" from their "abjected" zones. Then, the planning for progress and development is to model the "efficient teacher" as agent of change who fills the gaps and who provides learning opportunities for those located in unlivable spaces, which in fact related to the making of children as productive democratic citizens of the nation.

In asking the question, for example, "what kind of learning experiences *will* prepare students for the demands of the twenty-first century?" (Smith & Stein, 2011, p. 1, italics original), the concern here is not merely an issue of teaching and learning mathematics, but more about the being prepared when the demographic shifts happen where "'minorities' will constitute the majority of school children by 2023" (NGA, 2008, p. 14). "Being prepared" does not only include economic competition in the global sphere, but also embraces a desire to ensure the collective belonging of the "minorities" to the nation; a desire to maintain technological and scientific leadership of America across the world; a desire to conserve the "culture" and "democracy" inherited from the founders of this country. Although it might seem like the "minorities" are asked to serve these desires, they are also becoming part of this imagined yet calculated future as mediated by the collective hope to achieve "a society that is mathematically capable". So, "being preparedness" is an anticipatory action to secure, conduct, discipline and normalize particular forms of life in the contemporary liberal democracies (Anderson, 2010).

Being prepared for unknown futures also operates as an ordering practice in the making of "efficient" teachers while modeling the effective pedagogy. While the "necessity" of correcting the "inequalities" between fictitious categories requires an "additional support" to ensure access and equity for all, the desire to include who is excluded before simultaneously becomes an ordering and normative set of practices to make particular kinds of teachers. “Effective” teachers also “need help to understand the strengths and needs of students who come from diverse linguistic and cultural backgrounds, who have specific disabilities, or who possess a special talent and interest in mathematics” and they have to be prepared to accommodate differences not only continuously accumulate knowledge and strategies for teaching for diversity but also “they need to understand and confront their own beliefs and biases” (NCTM, 2000, p. 14).

Teachers are to become “engineers of the learning environment” (Stein, et. al., 2008, p. 315; NCTM, 2014) rather than the dispenser of the mathematical knowledge. To make the classroom discourse productive and the community effective, as modeled in the research literature, teachers need to “offer instructional support” to children “who appear to be on the verge of implementing a unique and important approach to solving the problem, but who need some help *to be able to actually achieve that and effectively share it with their classmates*” (Stein, et. al., 2008, p. 328, my italics). Then, what the “instructional support” offers, in fact, is not related to mathematical idea that the child is producing or constructing. Instead, it is the support for an effective sharing of ideas.

While the “effective” teachers are the subjects or the agents of this communicative discourse of the classroom where they “know how to ask questions and plan lessons that reveal students’ prior knowledge; they can then design experiences and lessons that respond to, and build on, this knowledge” (NCTM, 2000, p. 16), this agency becomes normative category that locates teachers in in a continuum from “novices” to “experts”, which signifies hierarchical “levels” of classroom discourse (NCTM, 2014). That is to say, “novices” are the teachers who *need* a “model of the five practices” enabling them to facilitate discussions and who are to become “experts” over time (Stein, et. al., 2008). Nevertheless, this "model" is never fully developed and does not have an end point "rather as a set of emerging and provocative ideas for rethinking how [to] prepare novice teachers" (Grossman, et. al., 2009, p. 274).

Being prepared for the uncertain futures requires a necessity to embody an effective pedagogical model that put teachers as objects of research, reform or intervention for progress and development. However, the concern for being prepared generates more. Transmutation of the discourse about teachers and teaching into "modeling", which signifies teaching as an ever-changing and never-ending product, does re-produce teaching bodies that are epitomized as adaptive, flexible, reflective selves to make "efficient" and "productive" decisions even in very complex systems and contexts. As NCTM (2000) reported, for example, "sequencing lessons coherently across units and school years is challenging. And teachers also need to be able to adjust and take advantage of opportunities to move lessons in unanticipated directions" (p. 15). While teachers are considered as the agents of their own classrooms where they make their own decisions, their agency, represented as "teaching practice", becomes a category to be mastered for the purpose of being prepared for unforeseen circumstances and complex systems. This paradox is considered to be solved by making the development of the teaching practices flexible, adaptive, consisting of ever-emerging ideas and not providing an exhaustive list to do or a recipe of effective teaching procedures. However, unanticipated directions are never allowed by the very practice of "anticipating what students might struggle with during a lesson and being prepared to support them productively through the struggle" (NCTM, 2014, p. 52). That is, while effective teaching requires "flexibility", "reflection", "ownership" and an exercise of teachers' own agency in productive ways, they are simultaneously regulated and controlled by a "continual effort to improve" (NCTM, 2000, p. 17). The improvement is both for the teachers themselves as "lifelong learners" who always need to be part of their professional development and for the children who need to further particular ways actions and participations. Although the continual effort to improve is part of the planning for "progress" and "development", it concurrently encounters with the ontological primacy of mathematics as an illusory "gatekeeper" category to order, make, classify, differentiate and normalize particular type of teaching and produces particular kinds of teacher who fits to that fictitious category.

Moving all students toward the development of important mathematical ideas might be regarded as a commonsensical aim of the mathematics education field; however, when it becomes an "anticipatory thought experiment", it allows a mechanism to exercise the relations of (bio)power that constitutes an economy of cognition as I have argued in the previous chapter. That is, if an "effective model of teaching" is to "facilitate discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments" (NCTM, 2014, p. 29), teaching becomes a war machine that allows the economy of cognition to operationalize by legitimization of discursive practices such as "mathematical needs", "democracy" or "mathematical mindsets" as argued above, becomes one of the fundamental mechanisms in the field of mathematics education as well as in the normalizing society to secure the power relations. The permanent "comparison" of students' approaches allows a competition, which produces racialized practices for all as the fittest can survive. In this mechanism, some are eliminated, excluded or they are corrected to be fit in these illusory categories.

4. Conclusion

The shift from “discovering the mathematical world” towards “mathematical modeling of the world”, as argued before, has shown the changes in the contemporary mathematics education practices while revealing a historical reiteration of making the self and the world based on representational premises. The constancy of these premises was not a philosophical deadlock; on the contrary, the shifts in practices have showed us how the field is continually contested, re-formed and never fully come back to exact same point. That is, the administrative machine of “math for all” does not entail an essence and a capacity of its own but becomes meaningful only if assembling with multiple discursive practices. The discipline and control mechanisms that organize the pedagogical and the social space make the technologies on and of the body both productive and subjected to the knowledge. “Mathematical standards” or “effective mathematics teaching” are taken as “enabling” practices for what children are able to do to achieve a society that is mathematically capable.

“Doing mathematics”, rather than “possessing mathematics”, also, reveals the changing practices as rationalized and legitimized by the contemporary mathematical needs, demand for democracy, intelligent and efficient citizenship and a collective desire towards bright mathematical futures. In this chapter, my aim is to explore how these discursive practices assemble with one another to make “school mathematics” possible while these practices are to fabricate particular subjectivities that shape and fashion conduct of children and teachers while creating (un)livable cultural spaces. Nevertheless, the pair of self-other does not constitute a dialectical relationship. On the contrary, these processes of fabrication and abjection are possible due to amalgamation of distinctions. The distinction between mathematically able bodies and their Others is far from to be placed binary categories; on the contrary, there is a topological relations to make the life of the collectives. It is a process of inclusive exclusion, in Agamben's (1998) terms, requiring a paradoxical belonging as signified in the statements like "math-for-all". While the mathematically able bodies are to become the desired subject in these statements, it simultaneously circulates as the body of knowledge of mathematics education field as materialized in everyday life.

Chapter VI

Conclusion

The risk of writing a concluding chapter might be to inscribe another "redemptive narrative". But I want to leave this dissertation, as my roadmap, with some final remarks. Without actually terminating, this space is for me to encounter with an in-between moment to build a nexus with avenues yet to come.

1. "Modeling": Contemporary Mode of Scientific Practice in Mathematics Education

The aim of this study was to explore the commonsensical practices in the mathematics education field. I wanted to re-think how "modeling" is becoming as a way to reason about world, children and teachers/teaching in the contemporary curriculum and research practices. Tracing the multiplicities of discursive-material practices since 1950s revealed the entangled relationships with "modeling" as a mode of enactment about world, children and teachers in the present. While the language of "modeling" provides flexibility and allows multiple possibilities for the actors of schooling and educational research, it produces cultural norms (e.g. anticipation, monitor, self-assessment or profiling) that simultaneously constitute and restrict the formation of the selves, locations and cultures. Nevertheless, modeling revealed a capacity to act upon people not because it has an intrinsic power but because its exterior relations such as historical and cultural narratives, socio-political hopes and desires, where it entangles with the multiple discursive lines, practices and segments (Deleuze & Guattari, 1987). That is, "modeling" becomes the mode of "scientific" practice circulating across the field of mathematics education that make a particular mode of thought possible to manage and control messy realities as reflected in the practices shifting from rationality to rational choice, from one single product to process, from common ground to consensus, from tracked spaces to tractable common space, from isolation to communication. Reconfiguring mathematics education as *a model-driven science,* nevertheless, produces a regime of practices that govern self and society based on representational premises that are in fact distant from the reality.

While there is a persistent desire in the mathematics education research to provide conceptual tools to understand complex, dynamic, situations and systems that continually can adapt to diverse circumstances of the 21st century (Lesh & Sriraman, 2010), "modeling" in mathematics education reveals a regime of practices that establish certain and prescriptive futures in the name of security, progress and development through producing socio-psychological truths about children and teachers, which simultaneously legitimize this regime. Taking uncertainty into account, being responsible to the diverse interest and needs and embodying the various forms of relatedness become non-sense but the exercise of power relations. Although it is argued that most of mathematics education research is not "model-driven" and is based on ideologies that tends to be like religion and orthodoxy (p. 142), it is my contention that modeling in mathematics education establishes a grand narrative that shapes and fashions particular kinds of people with the necessity of faith in "science", not only with numbers but also with other forms of qualitative data. *Model science*, which makes up people and the world, as my analysis makes visible, are the re-enactments of civilization-colonization processes and bounded with Enlightenment reason and rationality and embodied particular religious ethics and their faiths. Hence, the practices that respond to the demand of breaking the "religiosity" of "ideology-driven" research and to transform into a "scientific" and a "model-driven" one remains ironic.

While I limit this study with the mathematics education practices, “modeling” as a mode of thought and practice also appears elsewhere. In the field of science education, for instance, “scientific method” is replaced by the “model-based inquiry”, which is argued as a new and shifting paradigm of school science investigations (Windschitl, Thompson & Braaten, 2008). Having said that, I can easily speculate that the discursive analysis of school-science practices would look similar to what I have argued so far since the subject matter in these discourses rests on an illusion.

Also, with the push of international examinations and the impetus of educational governance in the transnational sphere, the “big-data” studies in educational research proliferated where the “traditional” statistical procedures, such as t-tests or descriptive statistics, are replaced by structural equation *modeling*, hierarchical linear *modeling* or Rasch *models* in order to take more variables into account and to “explain” and “compare” messy educational realities across the globe. Although these international examinations are part of the discursive assemblage of school mathematics as a legitimizing practice, as I examined a little in the previous chapters, I will leave the particular discussion of modeling practices of “big-data” and “new” algorithmic procedures for other avenues.

2. Beyond Good and Evil: Mathematically Able Bodies As a Style of Thinking

The three chapters on “mathematics”, “ability” and “body” is not a separation of the field(s) as distinct of entities, rather they collectively contemplate the planes of intensity in the formation of regime of practices across two historical moments of mathematics education in the United States. My intervention in these power-knowledge relations is to map mathematically able bodies in the discursive field of mathematics education as an analytical move in the constitution of the body of knowledge, as mathematically able bodies, across two moments. That is, I have taken up mathematically able bodies as a theoretico-methodological style of thinking to investigate the formation of knowledge in modern mathematics curriculum in a particular space-time and as an event that makes visible the continuities and discontinuities in the power-knowledge relations in the field of mathematics education. The historical encounter with the "mathematics", "ability" and "body" is not an effort to take these notions for granted as an established category. Nor is it an identification and recognition of particular topics. On the contrary, my intention was to dissipate what has been taken as natural and unquestionable to form new compositions with a different style of reasoning.

Mathematically able bodies is not stable identity or a fixed body but it is performative, in which it has no ontological status without materialized acts constituting its reality, in the fluid webs of the discursive assemblage of school mathematics. Mathematically able bodies is not being abled by mathematics itself, rather become abled with their entanglement with the discursive assemblage of school mathematics as a cultural-historical practice that makes up people and the societies. It is a historical construct that moves from different layers in a manner that one builds and relates the one before; but extends and develops so there keeps transforming itself but with new technologies and tactics looping back not to the original points yet maintaining the continuities that could be explained in a spiral set of connections. It is a style of thinking that brings onto-epistemological framework of discursive assemblage of school mathematics into question and redirects attentions on the epistemological and the political anxieties that produces the ways of being, acting and participating for people yet entangled with their own local histories.

I think it would not be dangerous to suggest using mathematically able bodies as a style of thought to explore the formation of body of knowledge in the field of mathematics education in a specific time-space. It might be interesting to make visible what is disassociated and associated in these histories not to reach the origin of knowledge in mathematics education field but to re-think the illusory boundaries and to re-write entangled histories that (dis)connect people, cultures, places and times.

3. Making of Differences: The "Reason" of Mathematically Able Bodies

In the field of mathematics education, everybody wants to make a change: Teachers, researchers, students, parents, policy makers, and curriculum reformers to name a few. Nevertheless, the change is usually practiced as a process of identification and representation of the subjects who is supposed to make a difference. Although the language is shifted towards "modeling" that takes the uncertainties into account and the field is moving towards inclusive practices such as "math for all", the promises of change retrofits the existing normative categories and regenerates exclusions. The different style of reasoning embodied in this dissertation, where the subjects are decentered, was to study processes of fabrications and abjections as the concrete results of differential material-discursive enactments and the strategic position of knowledge as a material practice to understand how the subject is constituted within power-knowledge relations (Popkewitz & Brennan, 1997). The processes of fabrication of particular subjectivities (i.e. decision maker, flexible thinker, adaptive human kind) enclose and abject others that are located in unlivable zones. Although two human kinds are assumed in these enactments, constitution of differences and demarcations is not a dialectic relationship but a historically constituted one with multiple entanglements. It is also a process of "inclusive exclusion", borrowing from Agamben (1998), requiring a paradoxical belonging to one another albeit a mechanism of differentiation, as signified in the statements like "math for all".

The comparative historical study of the two moments in the mathematics education practices is not to take "math for all" or "school mathematics" as an object or a thing to be replaced by something else but to locate some of the historical and cultural trajectories that enabled the circulation of "math for all" in mathematics education and how this discourse can become constitutive of what is "dangerous" or "good" in the particular time and space in the name of progress and development. School mathematics entangled with the societal hopes and fears produces cultural theses for the modes of living and fabricates able bodies that can function in modern societies in "civilized", "secure" and "happy" ways. Although the contemporary "inclusive" practices appear as a disagreement with the previous ones such as tracking or controlled psychological experiments, when I historicize, the "new" and "reformed" practices are the re-enunciation of Enlightenment reason and rationality, moral qualities of life, embodying Judeo-Christian ethics, and civilization-colonization processes. In each chapter, I have grappled with these questions not to prove that nothing changed at all but to make visible the continuities and discontinuities across two moments by simultaneously looking at the specific mathematics education practices of pre-post WWII, right after entering the curricula as a required subject (1930s-1945s) and of today (1980s-present). That is, there is something sticky in the "reason" of school-mathematics as an actor to govern masses while making the differences (Popkewitz, 2008). Yet, it never re-inscribe exactly same processes but spirally extends and makes the field of power relations more active and diffuse, targeting our lives and bodies in very material ways. The discontinuities, in fact, show the polymorphous characteristics of "knowledge" that is always questionable. While I refer mathematically able bodies as a body of knowledge that forms an event in a particular locale, it simultaneously becomes an ontological category that depicts the objects of research and subject of life on a hierarchy of values. Dismantling the legitimizing practices of this spatiotemporal configuration is a way to open up future multiplicities through releasing the subject from its onto-epistemological premises that regulate, order, differentiate and normalize (Butler, 1992). This performative practice is a strategy of change and is part of the formation of the political will in the mathematics education and society.

4. Limitations and Potentials of Studying Mathematically Able Bodies

The first half of the century for mathematics educators, as Kilpatrick (1992) argued, was a search for an identity to define their field where they found themselves with a strong allegiance to mathematics and psychology. Although there have been several instances and moves that depart from these two fields incorporating social, cultural and sociopolitical aspects (see Stinson & Bullock, 2012), the discursive-material analysis of contemporary practices reveals a spiral loop to the "previous" practices. Problematizing the progressive, accumulated and linear Time by historicizing the present, in this study, was to explore the limits of distinctions between previous and contemporary or traditional and reformed in a particular space. Then, the snapshots of two schools, which I have narrated in the beginning of this dissertation, are not a change in the epistemological premises that constitute objects in the mathematics education field but a historical re-iteration of Enlightenment reason and rationality that makes a particular "mathematically able bodies" as regime of practices with no single origin to secure power relations.

One might ask: Is this study deadlocked with the paradoxes of freedom, democracy or agency? Is there no way to get out these assumptions that make the change impossible? Can't we be free at all? My answer is no, we are not stuck in these paradoxes. I would argue, on the contrary, the spaces for freedom and the potentialities of those paradoxes are embedded in the voids where each line hunches. The exploration of the limits of the mathematically able bodies does not give up the notions of freedom, democracy, action or movement. “What is given up”, as Popkewitz (2008) writes, “is the notion of planning people” that “stabilizes and fixes the boundaries of freedom” where resistance becomes the “continual pushing against the boundaries by historicizing what we are and what we have become” (p. 184). Then, the very act of historicizing mathematically able bodies can be considered as a form of resistance. Nevertheless, this act is not independent from the attitudes of Enlightenment and the commitments to freedom, democracy and justice.

The instabilities of the body of knowledge in mathematics education field have made this study possible. The kind of questions I have asked were concerned with the present and interested in interrogating the boundaries that limits freedom, democracy and what counted as “difference”. It would be equally ironic and paradoxical for me to offer future promises for mathematics education field in a study that critiques even the alternative possibilities, the “uncertain” prescriptions and models. Still the promise of this study is to evoke the question that asks what it means to live *a* life within *the* life that is historically organized and planned as mathematically able bodies.

> A life contains only virtuals. It is made up of virtualities, events, singularities. What we call virtual is not something that lacks reality but something that is engaged in a process of actualizations following the plane that gives it its particular reality (Deleuze, 2001).

So, the continual search to live *a* life always remains with us. But our lives are not outside of the histories that make us possible. I think Marx was quite right when he said, “men make their own history, but they do not make it as they please; they do not make it under self-selected circumstances, but under circumstances existing already, given and transmitted from the past”

(1869/1972, p. 10). The kind of historical register taken up in this study, nonetheless, goes beyond what the Marxist-Hegelian traditions offer. It has neither single origin nor predefined agents of revolution. But there are multiple and entangled cracks, historically embedded in these planes of intensity: Mathematics, ability and body. These planes with their cracks are of course not the whole story, cannot be, yet their ambivalent conditions form a movement for change.

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